Stochastic Arbitrage and the Valuation of Weekly Options: Theory and Empirical Evidence

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ABSTRACT

Using Stochastic Arbitrage, we present bounds for frictionless equilibrium option prices under jump diffusion asset dynamics. We apply these bounds to a large data set of Weekly S&P 500 index options from 9 to 1 days-to-maturity. We show that the observed prices are systematically below the frictionless equilibrium upper bound, particularly for several segments of the support of the index return distribution with highly liquid options. We confirm the overpricing with out-of-sample and model free tests. We argue that these bound violations are the reasons that several empirical S&P 500 index option studies failed to reconcile the index dynamics with the observed option market data. We attribute the overpricing to frictionless assumptions and to features of the intermediated market that are inconsistent with perfect competition assumptions.

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1. Introduction

Weekly S&P 500 index options (SPXW) are similar to standard monthly options (SPX), except they have a shorter life span, are PM-settled on their expiration date, and are expiring every day of the week. They are typically listed several weeks in advance. Since launch, Weekly options have grown to become one of Cboe's most-actively traded products. In 2021, a total of 345 million S&P 500 index option contracts were traded, with an average daily volume (ADV) of 1.4 million contracts. Among them, there were approximately 247 million SPXW contracts, with an ADV of more than 981,000 contracts, accounting for nearly 72 percent of total S&P 500 index option trading volume.[2]

Despite the significant trading volume and increasing role of SPXW in the index option markets, the investigation of its pricing did not receive the expected attention, unlike its SPX counterparts. To our knowledge, the only option valuation study dedicated at least in part to SPXW is by Anderson, Fusari and Todorov (AFT, 2017), who studied jump risk in short term S&P 500 index options, both SPX and SPXW. However, there are several empirical studies that used SPXW option data. Constantinides, Czerwonko and Perrakis (2020) apply to a sample of SPXW options an algorithm that can detect mispriced index option portfolios in the presence of frictions, but do not model prices of individual options. Chatrath *et al* (2015) examine differences in information shares between SPX and SPXW and find that implied volatility is substantially higher in SPXW. Chatrath *et al* (2019) investigate the behaviour (betting-time effect) of winners and losers in financial markets using intraday data of SPXW call options. Since AFT seems to be the only study dealing with pricing of SPXW options, it will be the main frame of reference for our results.

The No Arbitrage Equilibrium (NAE) theoretical framework used in most option valuation models including AFT is based on asset pricing theory in frictionless markets, in which the option values are expectations of the payoffs, with the underlying dynamics transformed to become risk neutral. Such a transformation implies that the return distributions are multiplied by a pricing kernel, for which various forms are available theoretically. Nonetheless, several studies do not introduce an explicit pricing kernel, use only option market data, and rely very little on the dynamics of the index return distribution. They implicitly assume that the frictionless equilibrium is consistent with the observed option data, which is not frictionless. In most empirical studies option data are transformed to conform to the frictionless markets assumption, discarding in-the-money (ITM) options and replacing them via put-call parity with the corresponding out-of-the money (OTM) options. They use the midpoint of the observed bid-ask spread of the options as representing the frictionless price.[3] With such transformed option data they extract the

[2] https://ir.cboe.com/news-and-events/2022/04-13-2022/cboe-add-tuesday-and-thursday-expirations-spx-weeklys-options
[3] See p. 1345, footnote 8, and Appendix D, p. 1374 in AFT.

parameters of the risk neutral frictionless process, which is fitted to the entire option cross section. In AFT this fitting is extremely flexible, insofar as it uses a semi-nonparametric approach that only imposes weak restrictions on the jump distribution. It allows for time-varying risk neutral jump amplitude distributions with daily calibration. In spite of this flexibility, however, they have trouble in identifying models that can adequately fit both right and left tails of this risk neutral distribution; see pp. 1354, 1356, 1358, 1364, etc. They conclude (p. 1370) that "there is a sharp separation between the dynamics of the actual jump risk and its pricing," and mention the need to identify the economic forces that can rationalize their results.

In this paper we achieve the following objectives. (a) We show that the problems encountered by the AFT NAE models stem from applying an option valuation model developed for frictionless markets to a market that is emphatically not frictionless, neither in trading index tracking funds or index futures nor, a fortiori, in trading options. (b) Using an extended SPXW dataset, we demonstrate that even in the very liquid moneyness regions, put-call parity does not hold even approximately.[4] Thus, we examine separately the partially segmented markets for call and put options. (c) We find that there are systematic differences between frictionless option valuation and observed option market data, varying by degree of moneyness of the options. (d) We apply the stochastic arbitrage (SA) approach to map the regions in the option market where the frictionless option values consistent with the observed index dynamics should lie. The flexibility of the SA approach allows this mapping at the level of individual options, rather than for an entire cross section, as in NAE models. We demonstrate that for large segments of the support of the index return distribution there is a complete disconnect between these mapped regions and the corresponding observed bid-ask spreads in our sample. (e) Last, we test rigorously whether this observed inconsistency of the option market data with the theoretically correct frictionless option bounds implied by the underlying returns can lead to profitable option trading. Our out of sample statistical tests demonstrate that the observed option data can produce large risk adjusted profits *provided* trading is frictionless, thus validating the SA bounds as identifiers of correctly priced options in a frictionless world.

To obtain Weekly option bounds we apply the stochastic arbitrage (SA) approach, formerly known as stochastic dominance. In this approach we compare two generic identical risk averse investors. One of them holds the index and the riskless asset, hereafter termed the index trader (IT). The other one, the option trader (OT), adds a zero net cost option position to the IT holdings. OT should not dominate IT to the second degree. This absence of dominance creates a region, with upper and lower bounds, within which the risk neutral option prices should lie. These bounds are the tightest intervals for the frictionless option prices to

[4] In our sample of end-of-week SPXW options, the put volume is about 1.8 times that of the call volume, and this ratio is reasonably persistent from 9 days-to-maturity until 1 day-to-maturity. See also Constantinides and Lian (2021) on the partial segmentation of the markets for puts and calls.

avoid risk-adjusted positive expected returns from the zero net cost portfolios. The SA approach uses only the index return dynamics, without involving the option market data. It is, therefore, uniquely suitable for the separation of the frictionless equilibrium option values from the option market data that is certainly not frictionless and requires separate modelling.

A frictionless equilibrium in the option market should not allow IT investors to realize positive risk adjusted profits by adding a zero net cost short or long option position to their holdings while trading in the frictionless market. A corollary of this condition is that frictionless option values can be extracted from the observed option market data bid-ask spread midpoint if and only if that midpoint is situated within the SA bounds for the entire support of the return distribution. This happens seldom in our data as the market overvalues both calls and puts with respect to the SA bounds across a wide range of moneyness.

If frictionless option values cannot be extracted from a portion of the support of the return distribution, then there are frictionless trading strategies that generate risk adjusted profits for the IT investor. *Any* risk averse investor holding the index and the riskless asset will increase her expected utility by adopting such a strategy. In our data, this is confirmed rigorously by genuine out-of-sample and model-free tests. These tests compare the realized aggregate payoffs of the adopted strategies plus the returns of the IT portfolio, the OT time series of portfolio returns, to the IT returns. They allow inferences about the populations from which both time series were drawn, thus confirming or rejecting the mispricing of the bounds violating option(s).[5] To our knowledge, the application of these tests to frictionless markets is novel. It requires the modelling of the IT investor portfolio, which is available explicitly only for investors of the Constant Relative Risk Aversion (CRRA) category but can be adapted to any value of the risk aversion parameter.

The SA approach also confirms the AFT-conjectured independence of the pricing of diffusive volatility and jump risk, represented by at-the-money (ATM) and OTM options respectively, by showing that only one of them is consistent with the SA bounds. We present empirical evidence that the bid-ask midpoint observed in the option market is an adequate representation of the risk neutral distribution supported by the index return dynamics *only* at the left tail of the distribution. By contrast, the option market overprices the highly liquid region around ATM for both calls and puts, with the admissible option prices lying way below the observed bid quotes for a large majority of the traded options. This result holds for all days-to-maturity (DTM) of the SPXW options from 1 to 9 days. In terms of implied volatility, the average difference between upper-bound implied IV and the Black-Scholes IV is at least 3% in annualized terms, for both ATM calls and ATM puts and for all DTM categories. Such a difference implies that the market quotes for ATM

[5] See Constantinides *et al* (2011). The tests' hypotheses verify whether risk averse investors *unanimously* prefer the time series of OT portfolio returns over IT portfolio returns.

options exceeds by more than 20% the maximum frictionless equilibrium option values that can be supported by the index dynamics.

We verify that these ATM options are overpriced by out-of-sample SA tests that short the options at their bid price and close the positions at the SA bound by the next day in frictionless trading. We show that this strategy generates risk-adjusted profits to OT trader with the profits disappearing if the positions are closed at the prevailing end-of-day ask prices. In fact, our results show that more than 50% of the observed bid-ask quotes for Weeklies are mispriced in frictionless markets with respect to the underlying index dynamics. Further, the consistency of index market data with the SA option bounds is strongly dependent on the option moneyness, indicating clientele effects that attract separate types of investors for ATM versus OTM options. This seems also to be implied by the comments of AFT (p. 1336), that ATM and OTM options are focused separately, respectively on the spot volatility and the jump component dynamics.

To our knowledge, this is the first empirical application of SA to frictionless markets, although it has been applied to markets with frictions. This approach was originally developed for frictionless markets in discrete time by Perrakis and Ryan (1984), Ritchken (1985), Levy (1985), Perrakis (1986), and Ritchken and Kuo (1988). It has been extended theoretically and shown to converge in continuous time either to a single value in the cases of diffusion or stochastic volatility or to two distinct boundary distributions when the diffusions are mixed with independent jumps in Ghanbari *et al* (2021) and references therein. In all cases, SA restricts the relevant universe of traders to those who are risk averse and hold the index and the riskless asset, an assumption that is also consistent with several NAE-based option valuation models.[6]

Our SPXW results are consistent with previous theoretical and empirical research that recognizes market frictions, such as Constantinides and Perrakis (2002, 2007), Constantinides *et al* (2011), Constantinides, Czerwonko and Perrakis (CCP, 2020), Post and Longarela (2021), and Arvanitis, Post and Topaloglou (2021). In CCP the one-week maturity option portfolios selected by their algorithms generated major risk adjusted expected excess returns in-sample, that were confirmed on the realized ex-post series of returns. Thus, if the options are mispriced in the market with frictions in a single period buy-and-hold model, then they are a fortiori mispriced in a continuous time frictionless model, with the excess returns augmented by the bid-ask spread of the chosen portfolios. More surprisingly, our SPXW results may also be consistent with the findings of Barone-Adesi *et al* (2020), who work in the frictionless market in the spirit of the NAE models. They verify the monotonicity of the pricing kernel by using the proportion of deep OTM options to adjust the joint extraction of the *P*- and *Q*-dynamics.

[6] See, for instance, Heston (1993), Wiggins (1987), and Heston and Nandi (2000). Note also that such index holders form a large and increasing share in the economy, as documented by Bogle (2005) and Charles (2017).

On the theoretical side, there are few studies that model transaction costs in the option market or consider the relation between the frictionless option values and the bid-ask spread in the market. An early such study was Jouini and Kallal (1995), who formulated equilibrium models for dynamic securities markets with bid-ask spreads. This pioneering study showed (p. 188) that for the "correct" or arbitrage free pricing of options in both the frictionless market and in the presence of frictions, the frictionless price should lie within the bid-ask spread. A corollary of this result is that the pricing kernel should pass through the option bid-ask spread in the entire option cross section. This was also confirmed theoretically by Constantinides, Jackwerth, and Perrakis (2009), Beare (2023) and Post and Longarela (2021). These studies showed that their mispriced option portfolios were due to the fact that there was no monotone pricing kernel passing through the bid-ask spread in most of their option market cross sections; the same is true for our data.[7]

Our results also provide some answers to the big question that AFT raise, about the identification of the economic forces that can rationalize the different behaviour of left and right tail of the risk neutral distribution. We fully agree with AFT that it is economic forces, not estimation technology, that are responsible for the impasse. Economic forces, however, manifest themselves through markets, where the equilibrium is established through supply and demand curves and possibly market power. The only market here is the option market, which is intermediated and universally assumed to be perfectly competitive. This assumption has never been tested empirically but it should be treated with suspicion, certainly for short maturity options and possibly for all index options. In the last section we present evidence that disproves it and motivates further research that transcends the scope of this paper.

The next section reviews jump diffusion option valuation and presents the derivation of the SA option trading bounds applicable to this paper, as well as the out-of-sample tests that verify whether the bounds are identifiers of mispriced options. Section III estimates the bounds for an extended data set of SPXW options and demonstrates via out-of-sample tests that the mispriced zero-net-cost option portfolios provide risk-adjusted excess returns to their holders in the frictionless market, but not in the presence of frictions. Section IV concludes.

2. The Frictionless Option Bounds and their Violations

Jump diffusion models of index returns were first introduced into option valuation by Merton (1976), as mixtures of constant volatility diffusion with independent Poisson jumps with lognormal amplitude. These dynamics were subsequently extended by Kou (2002), Kou and Wang (2004) and Fu *et al* (2017). Nonetheless for the purpose of valuing options these dynamics had to be transformed into risk neutral

[7] In fact, the mispricing was in the economy with frictions, and it held a fortiori in the frictionless economy; see also Constantinides, Czerwonko and Perrakis (2020).

processes through equilibrium models, which proved more challenging. Merton (1976) had shown that the presence of jumps made the markets incomplete and there was a need for additional assumptions over and above no arbitrage. His option valuation model is only applicable when the jump risk is unsystematic and not priced. Equilibrium models such as Bates (1991) and Amin and Ng (1993) used the device of a representative CRRA investor, whose RRA entered into the risk neutralization expressions. Since this proved to be too restrictive for empirical work, it was abandoned on behalf of extracting empirically the risk neutral distribution from observed option market data, as in AFT.

The SA approach represents an alternative approach to risk neutralization, which derives bounds within which the admissible risk neutral option prices should lie. These SA bounds are derived by considering the risk averse IT investor holding the index and the riskless asset, who adds to her holdings a zero net cost portfolio containing a long or short option position and becomes the option trader or OT. The option prices should be such that OT returns would not dominate IT returns in the second degree at any time interval to option expiration, a condition that translates into upper and lower bounds for the option. The bounds are consistent with a monotone decreasing pricing kernel. Any option price outside the bounds implies a stochastic arbitrage opportunity. The bounds are derived recursively, starting from the time $T-\Delta t$ before the option expiration at *T*, and applied to any time interval $(t, t+\Delta t)$ within the life of the option. The limiting form follows immediately as $\Delta t \rightarrow 0$. A key issue is the specification of the index return dynamics, to which we now turn.

Since we do need tradable entities, the dynamics of the index can be extracted from a tracking fund, as we assume in this section, or from a futures contract that matures with or after the option, as we illustrate in our online appendix when we introduce transaction costs. We follow AFT (eq. (1), p. 1348), who assume that the index returns are stochastic volatility mixed with jumps (SVJ), but also state that for such short maturities as in SPXW the volatility can be taken as constant. Hence, we use physical parameter estimates under jump diffusion and assume a total volatility that is time-varying between cross sections but constant for each cross section. That volatility is estimated from adjusting the observed VIX index for premium. As shown in Perrakis (2019, pp. 220-223), the volatility estimates from the adjusted VIX were excellent forecasts of the ex-post observed realized volatility. We verify them in our robustness checks.

For a tracking fund, let I_t denote the value of the index at *t*, μ the instantaneous mean return assumed greater than the riskless return r, q the (assumed constant) dividend rate, λ the jump intensity, κ the expected log jump amplitude minus 1, and σ_t the diffusion volatility, which is assumed to remain constant in each cross section. Let $[t,T]$ denote the interval to option expiration T and K is the option strike price. The index dynamics then become

$$\frac{dI_t}{I_t} = (\mu - q - \lambda\kappa)dt + \sigma_t dW + (j-1)dN\,. \tag{2.1}$$

Hereafter we ignore the dividends that do not enter into the risk neutral dynamics. The discretized version of (2.1) is, setting the ex-dividend return $z_{t+\Delta t} = \dfrac{I_{t+\Delta t} - I_t}{I_t}$

$$z_{t+\Delta t} = \begin{cases} [\mu - \lambda\kappa]\Delta t + \sigma_t \varepsilon\sqrt{\Delta t} & \text{with probability } (1-\lambda\Delta t) \\ [\mu - \lambda\kappa]\Delta t + \sigma_t \varepsilon\sqrt{\Delta t} + (j-1) & \text{with probability } (\lambda\Delta t) \end{cases}, \tag{2.2}$$

where $\varepsilon \sim N(0,1)$. We assume that the jump amplitude j is a *truncated* lognormal such that $j > j_{\min} \equiv \underline{j}$, which is common to all cross sections, consistent with the assumptions of the jump diffusion SA bounds. We adopt for $\underline{j}$ a value of a 20% drop in the index in a single jump. This is very conservative as it overestimates the left tail risk, given that the maximum *daily* drop ever observed in the S&P 500 index was 22.61% on October 19, 1987. As it turns out, this choice biases upwards the SA upper bound, as discussed further on in our empirical work and robustness tests. We have $\kappa = \exp\left(\mathrm{E}\left[\ln(j) \middle| j \geq \underline{j}\right]\right) - 1$. With this specification the total variance of the ex-dividend index return till option expiration is given by the following expression, which is observable at every cross section.

$$\mathrm{Var}\left[\ln\frac{I_T}{I_t}\middle| j \geq \underline{j}\right] = \left[\sigma_t^2 + \lambda\left(\left(\mathrm{E}\left[\ln(j)\middle| j \geq \underline{j}\right]\right)^2 + \left(\mathrm{var}\left[\ln(j)\middle| j \geq \underline{j}\right]\right)\right)\right](T-t) \tag{2.3}$$

The constant instantaneous mean μ can easily be varied or set proportional to variance or to volatility, with very little effect on the results. The derivation of the SA bounds stems from the Euler discretization of the diffusion component, in which the dynamics are as in (2.2) but with the random term $\varepsilon \sim F(\varepsilon),\ \varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}],\ E[\varepsilon] = 0,\ E[\varepsilon^2] = 1,\ \underline{\varepsilon} < 0 < \overline{\varepsilon} < \infty$. These dynamics are the convolution $P_t(z_{t+\Delta t})$ of the time-dependent Euler-discretized diffusion with the independent jump component. As $\Delta t \to 0$ the discretized index dynamics tend to (2.1).

This discretization is applied to the general expression for the frictionless SA bounds, which holds for any distribution of the random return $z_{t+\Delta t}$, with $\mathrm{Min}\{z_{t+\Delta t}\} \equiv z_1$, $e^{r\Delta t} = R = 1 + r\Delta t + o(\Delta t)$ and

$E_t[z_{t+\Delta t}] \equiv \hat{z}_t$. Note that z_1 corresponds to $j_{\min} = \underline{j}$ as $\Delta t \to 0$. This yields the following risk neutral dynamics $U_t(z_{t+\Delta t})$ for the upper bound[8]

$$U_t(z_{t+\Delta t}) = \begin{pmatrix} P_t(z_{t+\Delta t}) \text{ with probability } \dfrac{R-(1+z_1)}{\hat{z}_t - z_1} \\ 1_{z_{t+\Delta t}=z_1} \quad \text{with probability } \dfrac{1+\hat{z}_t - R}{\hat{z}_t - z_1} \end{pmatrix}. \tag{2.4}$$

At the limit this expression yields, for $z_{t+\Delta t}$ given by (2.2)

$$\begin{gathered} \frac{dI_t}{I_t} = \left(r - \left(\lambda + \lambda_{Ut}\right)k^U\right)dt + \sigma_t dW_t^Q + \left(j_t^U - 1\right)dN_t^Q,\ \lambda_t^U = \lambda + \lambda_{Ut},\ \lambda_{Ut} = -\frac{\mu - r}{\underline{j} - 1}, \\ j_t^U = \begin{cases} j & \text{with probability } \frac{\lambda}{\lambda+\lambda_{Ut}} \\ \underline{j} & \text{with probability } \frac{\lambda_{Ut}}{\lambda+\lambda_{Ut}} \end{cases},\ E\left[j_t^U - 1\right] = \kappa^U = \frac{\lambda}{\lambda+\lambda_{Ut}}\kappa + \frac{\lambda_{Ut}}{\lambda+\lambda_{Ut}}(\underline{j}-1) \end{gathered} \quad . \tag{2.5}$$

An SA lower bound also exists for this jump diffusion process, which is not shown since it is rarely violated in our empirical applications.

Equation (2.5) is sufficient for the estimation of the upper bounds of the options in a given cross section, which are expectations of the corresponding payoffs. These bounds can be applied individually to every option. If there is an option, whose observed bid quote lies above the upper bound then this option is obviously overpriced in the frictionless world. The rest of this section is devoted to the design of strategies to exploit this overpricing for both calls and puts, which stem from the proof of the derivation of the risk neutral dynamics (2.4) in Perrakis (2019, pp. 23-27).

Suppose we observe at time *t* a call option in a cross section with maturity *T*, whose bid price lies above the SD upper bound, or $C_{bt}(I_t, K, T) > \overline{C}(I_t, K, T)$. The strategy consists in shorting one call per unit index at the bid price and allocating $\beta_t C_{bt}$ and $(1-\beta_t)C_{bt}$ in the riskless bond and the index, respectively. This position is closed at time $t+\Delta t$ at a price equal to the call upper bound $\overline{C}(I_{t+\Delta t}, K, T)$, which is derived from the discretized Q-dynamics in (1.4) for all risk averse traders. At time *t,* the allocation β_t is chosen so that at the lowest value of the return $z_{\min,t+\Delta t} \equiv z_1$, that corresponds to the left tail of the jump amplitude, the portfolio payoff from *t* to $t+\Delta t$ will be zero, implying the optimal allocation at time *t* would be

[8] See Perrakis, (2019, pp. 23-27), and Ghanbari, Oancea and Perrakis (2021, pp. 252-254).

$$\beta_t = \frac{\bar{C}(I_t(1+z_1),K,T)-(1+z_1)C_{bt}(I_t,K,T)}{RC_{bt}(I_t,K,T)-(1+z_1)C_{bt}(I_t,K,T)} \equiv \beta_t^*. \tag{2.6}$$

At option maturity *T*, the cumulative allocation would be equal to $C_{bt}\left[R^{T-t}\beta_t^* + (1-\beta_t^*)\prod_{\zeta=t}^{\zeta=T}(1+z_{\zeta+\Delta\zeta})\right]$, from which we must subtract the cumulative proceeds of the closed short call to find the OT payoff.

For an overpriced put option, at time *t*, we write a put option at its bid price P_{bt}, short $\beta_t I_t - P_{bt}$ unit of index with $\beta_t < 1$, and invest $\beta_t I_t$ in the riskless asset. The portfolio payoff at $t+\Delta t$ is $\beta_t I_t R - [\beta_t I_t - P_{bt}](1+z_{t+\Delta t}) - P(I_t(1+z_{t+\Delta t}),K,T)$, whose lowest value is when the put is at its upper bound at $t+\Delta t$, or $P(I_t(1+z_{t+\Delta t}),K,T) = \bar{P}(I_t(1+z_{t+\Delta t}),K,T)$. This payoff is clearly increasing in the put bid price P_{bt} for every β_t. At the lowest value of the index return support $z_{t+\Delta t} = z_1$, the payoff should be nonnegative, implying that the optimal allocation at time *t* is

$$\beta_t \geq \frac{\bar{P}(I_t(1+z_1),K,T)-P_{bt}(1+z_1)}{I_t[R-(1+z_1)]} \equiv \beta_t^*. \tag{2.7}$$

Setting β_t at its optimal level at time *t*, the expected portfolio payoff at $t+\Delta t$ is $\beta_t^* I_t R - E_t[(\beta_t^* I_t - P_{bt})(1+z_{t+\Delta t}) - \bar{P}(I_t(1+z_{t+\Delta t})),K,T]$. This expectation should be 0 at $P_{bt} = \bar{P}(I_t(1+z_{t+\Delta t}),K,T)$, which is verified using (2.4). Hence, it is positive for $P_{bt} > \bar{P}$, implying a positive put portfolio payoff when the put position is closed at $\bar{P}(I_{t+\Delta t},K,T)$. As with the overpriced calls, the excess returns at *T* would be $P_{bt}\prod_{\zeta=t}^{\zeta=T-1}(1+z_{\zeta+\Delta\zeta}) + \beta_t^* I_t[R^{T-t} - \prod_{\zeta=\tau}^{\zeta=T-1}(1+z_{\zeta+\Delta\zeta})]$, from which we subtract the cumulative proceeds of the closed short put to find the OT payoff. In our empirical applications, we liquidate the portfolio and close the option positions either at the SA upper bound for frictionless trading, or at the prevailing ask price in the presence of frictions.[9] Hence, each overpriced call and put options will contribute the following corresponding quantities

[9] Note, however, that frictions were not taken into account in establishing the option portfolios, implying that the resulting profits under frictions, if any, are upper bounds in exploiting the observed frictionless SA bounds' violations.

$$
\begin{aligned}
&C_{bt}\left[R^{T-t}\beta_t^* + (1-\beta_t^*)\prod_{\zeta=t}^{\zeta=T}(1+z_{\zeta+\Delta\zeta})\right] - R^{T-(t+1)}\begin{pmatrix}\overline{C}(I_{t+1},K,T) \text{ for frictionless} \\ C_{a,t+1}(I_{t+1},K,T) \text{ with bid-ask spread}\end{pmatrix} \\
&P_{bt}\prod_{\zeta=t}^{\zeta=T-1}((1+z_{\zeta+\Delta\zeta}) + \beta_t^* I_t(R^{T-t} - \prod_{\zeta=\tau}^{\zeta=T-1}(1+z_{\zeta+\Delta\zeta})) - R^{T-(t+1)}\begin{pmatrix}\overline{P}(I_{t+\Delta t},K,T) \text{ for frictionless} \\ P_{a,t+1}(I_{t+1},K,T) \text{ with bid-ask spread}\end{pmatrix}
\end{aligned}
\tag{2.8}
$$

This work leaves open the determination of the equilibrium price for the frictionless options, for which the SA methodology only provides trading bounds. As we shall see in the following section, these bounds are quite wide at the tails of the distribution and can support many equilibrium values. The call and put bounds implied by (2.6)-(2.7) are consistent filters of overpriced options for all frictionless equilibria, but the SA tests must be tailored to a particular equilibrium. A full determination of the equilibrium price requires further assumptions and a much longer treatment that transcends the purpose of this paper. We discuss these issues briefly in the following sections.

3. Data and Empirical Results

3.1 Estimating the parameters of the index dynamics and the SA upper bound

For the empirical estimation of the S&P 500 index returns' *P*-dynamics we use ex-dividend daily returns over the period January 3, 1963, to December 31, 2010. The underlying return sample is chosen so that it has no overlap with the option sample.[10] During this period the average annualized return is 6.25%, the standard deviation of returns is 16.27%, and their skewness and kurtosis are -1.07 and 32.18 respectively. In robustness checks we also use daily returns from January 2, 1980, to December 31, 2010. For the intraday index price, we use the price provided by the Cboe reporting system.[11] For the dividend yield we use daily cash payouts obtained from Standard and Poor. For the interest rate, we use the 3-month constant maturity T-bill rate obtained from the Federal Reserve Economic Data.

We estimate the jump diffusion parameters for the S&P 500 index using the generalized method of moments (GMM) framework. The moments of the jump-diffusion dynamics with a constant intensity and truncated log-normal amplitude are shown in Ghanbari, Oancea and Perrakis (2021, Appendix B). The parameter estimates are as follows. The mean return and diffusion standard deviation are 7.07% and 14.97% respectively. The jump intensity is 0.26 per year and the mean and standard deviation of the truncated jump

[10] Although SPXW started trading in 2005, they were AM-settled until December 2010 and PM-settled thereafter. To prevent any bias due to the settlement mechanism, our option sample starts in January 2011.
[11] In studies that used earlier data, the intraday index price was found from the cost-of-carry relationship between the cash index and its futures price, due to poor reporting of the cash index. As of 2007, the quality of this reporting has significantly improved and there is no reason to use futures.

size are -3.22% and 10.07% per year. The statistical properties of our parameter estimates are in Table OA1 of the online appendix.

We use the adjusted VIX index as a proxy for total daily volatility of the index returns. We adjust diffusion volatility in every cross section but keep the jump parameters constant. The adjustment in VIX is equal to the mean difference between the VIX and the realized volatility from 1986 to the observation date. The amount by which VIX exceeds the realized volatility is relatively stable in our sample, with an average annual premium of 5.27%, with the maximum (minimum) value of 5.61% (4.96%) in September 2007 (April 2020). Both the VIX and the realized volatility of daily returns are measured in one-week intervals without overlap, with the latter quantity defined as the square root of 252 times the mean squared daily return. In our robustness checks, we also consider alternative specifications of the VIX bias with very little impact on our results.

From equation (2.3) we extract a diffusion volatility for each option cross section given the parameter estimates of the jump distribution and the adjusted VIX observations. The average diffusion volatility across all cross sections is 13.49% with the minimum (maximum) diffusion volatility of 2.55% (74.44%). From these P-parameters we estimate the upper bound on the option prices as the payoff expectation with the dynamics (2.5). These dynamics stay the same for all options in a cross section but obviously vary by cross section because of the changing diffusive volatility. An observed market option value is correctly priced in the frictionless world if and only if it lies below this estimated upper bound. As we show further on in this section, in such cases we can also estimate the price of jump risk under certain assumptions.

3.2 The option data

In this study, we only focus on the End-of-Week SPXW options, maturing every Friday. Although the sample data in the AFT study contains all short-term options in their estimations, including both SPX and SPXW, we do not include the SPX in our sample, since they have different market structure, settlement mechanism and Designated Primary Market Maker (DPM).

Our option sample contains end-of-week (Friday expiration) Weeklies from March 17, 2011, until March 17, 2022, for a total of 477 expiring Fridays and 3127 observation days. We track each SPXW option in its daily trading over 7 different maturities, 9, 8, 7 and 4 to 1 days to maturity, from Wednesday till the next Thursday. Unlike AFT that use end-of-day data from OptionMetrics, we collect option data from time-stamped intraday option quotes at 3 PM from Cboe. Our sample only contains options with nonzero trading volume and minimum bid price of 10 cents. Standard arbitrage violation filters across moneyness were also applied. The trading volume is obtained from the Cboe Market Data Express Open/Close database. This dataset shows daily buy volume, sell volume, and open interest, separately for different categories of market participants. We confirm that for each option in our sample the net daily buy volume across all market participants is zero. The summary statistics for selected maturities are in Tables 1-3, and the full dataset is shown in Tables OA2-OA6 of our online appendix. The tables also report the average estimated SA option bounds, the overlap of quotes and bounds, and the location of the bid-ask midpoint within the bounds, separately for calls and puts. The online appendix also shows option summary statistics across different trading days. Overall, the sample has 140,305 (194,470) call (put) option contracts with average implied volatility of 22.7% (23.4%).

Several general conclusions can be drawn from this comprehensive data set, which to our knowledge is presented here for the first time. First, the market data for call and put options do not support put-call parity even approximately. For all maturity categories, options with 9 to 1 DTM, the total volume is more than 80% higher for puts than for calls. For the three observation days with 9, 7 and 4 DTM, the put volume is more than 77%, 120% and 108% respectively higher. Second, the market makers (MM) hold a large and relatively stable share of about 50% of the trading volume in the intermediated market for both calls and puts in every DTM category. Such a level of volume certainly does not support the free entry of dealers into that market, assumed in the equilibrium model of Garleanu, Petersen and Poteshman (GPP, 2009, p. 4264).[12] Last, the MM's order imbalance, which plays an important role in establishing equilibrium in the intermediated market in GPP and also in Fournier and Jacobs (2020), is not reliable as a key variable for the very simple reason that it is highly unstable during the life of the option. Although its averages start at the low levels of 5.35% for calls and 1.37% for puts 9 days before maturity, they rise dramatically to 18.72% for calls and 22.56% for puts at 7 days to maturity, only to fall at 0.2% and 2.4% three days later. We discuss the frictionless equilibrium consistent with the SA bounds further on in this section.

3.3 Option market data and the SA bounds

Tables 1-3 provide summary statistics for the end-of-week SPXW call (Panel A) and put (Panel C) options with 4-, 7-, and 9- DTM, disaggregated into moneyness ($m_t \equiv K/I_t$) categories. The statistics include relative bid-ask spreads, trading volume, and the MM's exposures in terms of net buys across each moneyness category. These tables also show the relative positions of the SA option upper bound and the bid-ask quotes for calls (Panel B) and puts (Panel D). This relative position is represented by two metrics, the proportion of contracts for which the bid-ask midpoint lies between the SA bound, and the proportion of contracts in which the bid-ask spread lies entirely above the SA upper bound, the percentage of overpriced contracts. It also reports the percentages of cross sections in which 100% and more than 50% of

[12] Note, however, that this may have been true in the 1996-2001 period of that study's data.

the contracts have bid quotes above their SA upper bounds. The full data statistics including all DTM categories are in Tables OA2-OA5 of the online appendix.

The most striking result in these tables is the inconsistency of the intermediated market with the fundamental conditions that must exist in order to extract the frictionless option market prices. Panels B and D in Tables 1-3 and Table OA2 show the results of the comparisons of the SA option bounds with the observed bid-ask prices of the options. In all seven DTM categories and for a very large majority of the cross sections, both put and call prices are fundamentally inconsistent with the SA bounds. In particular, the bid-ask midpoint, the universally accepted proxy for the "correct" estimate of the equilibrium option price, lies within the SA bounds for only 24% of the observed calls and 39% of the puts. In addition, the observed bid-ask spreads lie entirely above the option upper bounds for more than 37% of the calls and 35% of the puts across all ranges of moneyness. The results are even more extreme for the subsample of ATM options. In what follows we explore in depth the economic reasons for these inconsistencies.

[Table 1 about here]
[Table 2 about here]
[Table 3 about here]

The inconsistency of the bid-ask spread with the SA bounds is very definitely related to the option moneyness. The observed option prices in the ATM zone of $m_t = [0.98, 1.02]$ are virtually entirely disconnected from the corresponding SA bounds, for both calls and puts in all three DTM groups. A small minority of the bid-ask spread midpoints lie within the SA bounds, and for a large majority of the cross sections most option spreads have no overlap with the SA bounds. Exactly the opposite holds in the deep OTM put option zone of $m_t = [0.86, 0.93]$ for the 9- and 7-day DTM groups, and $m_t = [0.90, 0.96]$ for the 4-day DTM group, where the bid-ask spread midpoint lies within the SA bounds for a majority of the options and across a majority of the cross sections. The tables show that the percentage of put contracts with the bid-ask spread midpoint within the SA bounds increases as we move from the ATM zone to OTM zones. For OTM and deep OTM calls there is overlap between bid-ask spread and option bounds for most cross sections, but the spread midpoint is not within the bounds.

These results explain fully the inability of AFT to extract satisfactory risk neutral dynamics and tail risk measures from the observed option market data, even after the daily structural calibration of a semi-nonparametric model combined with heavy data manipulation to transform it into a frictionless format. Given the non-overlap between underlying-implied option bounds and observed bid and ask quotes for a large segment of the support of the index return distribution, there is simply no option market data consistent with the return dynamics that can be extracted from the S&P data. This fact, combined with the Jouini-

Kallal (1995), Constantinides, Jackwerth and Perrakis (2009), Beare (2011, 2023) and Post and Longarela (2021) theory, implies that the frictionless SPXW options are mispriced, and the extent of mispricing varies with option moneyness. In the remaining part of this section, we focus our investigation on the two important issues implied by our empirical work. The first one is the verification of the mispricing of the options with respect to the underlying prices. We explore the ability of investors to set zero-net-cost portfolios to profit from the SA-identified mispriced options and then examine the statistical significance of returns to these portfolios. The portfolio strategies and their payoffs are described in equations (2.6)-(2.8) for both frictionless trading and in the presence of an option bid-ask spread; see footnote 9.

The second issue derived from the data in Tables 1-3 concerns the inconsistency of the observed option market prices with the frictionless equilibrium prices implied by the *P*-distribution, which need to be reconciled for statistical testing purposes. As discussed in the introduction, on theoretical grounds this reconciliation can only come from the option prices that are broadly consistent with the SA bounds. The only such "correctly" priced options are the deep OTM puts, for which as it turns out the flexible jump coefficients fitted by AFT (p. 1356) adjusted to give a good fit! By contrast, these flexible coefficients occasionally failed to fit the OTM calls, a failure that is also consistent with our own results. The market for call options cannot be reconciled with the SA bounds implied by the *P*-dynamics, since most of the observed prices' bid-ask spread midpoints fall above the SA bound. The fitting by AFT of the risk neutral coefficients tailored to every cross section but constant across moneyness categories obfuscates the strong moneyness effects observed in the unadulterated option market data and the *P*-distribution-based SA bounds.

Unlike the applications of these tests in the market with frictions, the format of the ex-post mispricing tests of the frictionless market must be tailored to some equilibrium consistent with the SA option bounds. By construction, these bounds hold for all risk averse investors, implying that all such investors can profit from shorting overpriced options, but the application of the strategies described in (2.6)-(2.8) depends on the risk neutral dynamics derived from the IT investor underlying index holdings, which in turn depends on her attitude towards risk. To our knowledge, explicit forms for the equilibrium risk neutral dynamics corresponding to jump diffusion *P*-dynamics are available only under the assumption of a representative CRRA investor. The next subsection describes this equilibrium which is consistent with the SA option bounds and the implied tests for overpriced options. We stress the fact that the SA upper bound does not depend on the representative investor belonging to the CRRA class and stems solely from the monotonicity of the pricing kernel, which in turn can come from any risk averse IT utility function. The demonstrations below filter out the representative CRRA investor(s) corresponding to a given equilibrium below the SA bound.

3.4 Frictionless equilibrium under a CRRA investor

An equilibrium based on the assumption of a representative CRRA investor is defined uniquely from the RRA parameter. With the RRA denoted by φ, the risk allocation of the jump diffusion dynamics takes the following form[13]

$$\mu - r = \varphi\sigma_t^2 + \lambda\kappa - \lambda^Q\kappa^Q, \ \ \lambda^Q = \lambda E_t(j^{-\varphi}), \ \kappa^Q = E_t[\frac{(j-1)j^{-\varphi}}{E_t(j^{-\varphi})}] = E_t[\frac{(j^{1-\varphi})}{E_t(j^{-\varphi})} - 1]. \quad (3.1)$$

This equilibrium relation varies by cross section because of the variations in the riskless rate and the diffusive variance, implying that the corresponding φ varies as well. Our empirical tests, however, require that the same IT investor trades in all cross sections. We describe below the derivation of such an investor's RRA.

The IT investor maximizes successively her expected utility by allocating all her wealth to risky and riskless assets over a horizon T' longer than option maturity, $T' > T$. The CRRA utility allows intermediate consumption before T' without any effect on the portfolio allocation. Nonetheless, we believe that the empirically most relevant IT investor set are portfolio managers. Such people are, to quote Shleifer and Vishny (1997), "highly specialized investors using other people's capital", implying that the profitability of their decisions is subject to periodic reviews. For this reason, we assume, for simplicity and without loss of generality, that there is no IT consumption until after the review period $T' > T$.

Define portfolio return from *t* to T' as $R_t^{T'} = \alpha_t \prod_{\tau=t+1}^{\tau=T'}(1+z_\tau) + (1-\alpha_t)R^{T'-t}$, in which case for an initial wealth W_t, we have $W_{T'} = W_t R_t^{T'}$. The first order conditions (FOC) for the maximization of $E_t[U(W_{T'})]$ with respect to the optimal allocation α_t^* are $\frac{E_t[U'(W_T)\prod_{\tau=t+1}^{\tau=T'}(1+z_\tau)]}{E_t[U'(W_T)]} = R^{T'-t}$, for the CRRA utility function $U(W_{T'})$. Solving and replacing we define $R_t^{*T'} = \alpha_t^* \prod_{\tau=t+1}^{\tau=T'}(1+z_\tau) + (1-\alpha_t^*)R^{T'-t}$, in which case the FOC become $\frac{E_t[(R_t^{*T'})^{-\varphi}\prod_{\tau=t+1}^{\tau=T'}(1+z_\tau)]}{E_t[(R_t^{*T'})^{-\varphi}]} = R^{T'-t}$, and the kernel is $\frac{(R_t^{*T'})^{-\varphi}}{E_t[(R_t^{*T'})^{-\varphi}]}$, independent of wealth. By construction of the SA bounds, the IT investor must hold at least one index unit in her portfolio to apply

[13] See Bates (1991). The proof is also shown in our online appendix.

the condition of non dominance of OT over IT in our out-of-sample statistical tests. The kernel then becomes, neglecting dividends, equal to $\frac{I_T^{-\varphi}}{E_t[I_T^{-\varphi}]}$. This constraint on IT imposes automatically an upper limit on φ, which also depends on the riskless rate.

With such a kernel, an equilibrium such as (3.1) is feasible with a representative CRRA investor if and only if the RRA satisfies (3.1) for all cross sections in our data. This is too restrictive, given the wide variations in the diffusive variance across cross sections. On the other hand, there is no reason for the IT investor to satisfy (3.1), provided that she holds one index unit in her optimal portfolio in all cross sections. Setting in the upper bound (2.5) of the risk neutral dynamics $(\lambda^U, \kappa^U) = (\lambda^Q, \kappa^Q)$, it is easy to see that $\mu - r = \lambda\kappa - \lambda^Q \kappa^Q$, implying that the entire risk premium compensates for the jump risk at the SA upper bound. This automatically defines the following upper bound on the IT investor RRA, which depends only on the jump parameter distribution.

$$\begin{aligned} &\varphi_{t,T}^{\max} = Max\left(\varphi \middle| \lambda^Q = \lambda E_t(j^{-\varphi}),\ \kappa^Q = E_t[\frac{(j-1)j^{-\varphi}}{E_t(j^{-\varphi})}] \right) \Rightarrow \\ &\mu - r = \lambda\kappa - \lambda^Q \kappa^Q \text{ for } \varphi = \varphi_{t,T}^{\max} \end{aligned} \quad . \qquad (3.2)$$

From (3.2), a unique SA-implied RRA coefficient $\varphi_{t,T}^{\max}$ can be obtained for every cross section, given the underlying jump return distribution parameters and the riskless rate. That rate, however, has fluctuated significantly over the period of our data, from almost 0 in 2011 to about 2.5% in early 2019 and again back to 0 in 2020. Consequently, the upper bound $\varphi_{t,T}^{\max}$ on RRA has fluctuated within relatively wide ranges, which are very similar in the three reference DTM groups of 9, 7 and 4 days. In decreasing order of DTM, we have $Min\{\varphi_{t,T}^{\max}\} = [6.20, 6.99, 6.79]$ and $Max\{\varphi_{t,T}^{\max}\} = [14.16, 13.98, 13.90]$.[14] Thus, the SA-implied RRA $\varphi_{j,t:T}^{\max}$, $j = 1, ..., n$ in each cross section j is within the corresponding $Min\{\varphi_{t,T}^{\max}\}$ and $Max\{\varphi_{t,T}^{\max}\}$ of its DTM. For our portfolio strategy an IT-OT trader who exploits the mispriced options must have the same RRA coefficient φ^* for all cross sections. We must, therefore, have

$$\varphi^* \leq Min_j[\varphi_{j,t:T}^{\max}],\ j = 1, ..., n\,. \qquad (3.3)$$

What about IT investors who are more risk averse that the ones satisfying (3.3)? Since the payoffs of the zero-net-cost OT portfolio strategies are risky, our intuition tells that the portfolio payoffs will be lower

[14] The full distributions of the implied RRA coefficients for all DTM from 1 to 9 are shown in Figure I of our online appendix.

when the RRA increases, and this turns out to be the case. Any CRRA investor who is more risk averse, with $\varphi > Min_j[\varphi^{\max}_{j,t:T}],\ j = 1,\ldots,n$, for some cross section *j*, will invest in only a fraction of an option contract in that cross section. In our online appendix, we show that this fraction is equal to the ratio $\frac{\varphi^{\max}_{jt,T}}{\varphi} \equiv \zeta_j < 1$, hereafter termed the risk preference multiplier (RPM).

3.5 Exploitation of mispriced options

To exploit mispriced options, we focus on the close to ATM options with moneyness range $m_t = [0.98, 1.02]$, where the SA bounds do not intersect with the bid-ask spread interval for a large majority of the observed option contracts. In particular, only 10% (11%) of the bid-ask midpoints for calls (puts) is within the SA bounds in this moneyness zone. Hence, with the given return distribution, a sustainable frictionless equilibrium is not consistent with the observed ATM option market data. This implies that there exist stochastic arbitrage strategies generating risk adjusted positive profits for any risk averse investor. In the remainder of this section, we illustrate these strategies for individual investors. Note that we focus on this narrow ATM zone as it contains the most liquid contracts with high trading volume, about 79% (47%) of the total trading volume for the calls (puts) in our sample of SPXW options.

The time series of returns from trading mispriced options (OT portfolio returns) is obtained by the following steps, separately for calls and puts. (1) Filter overpriced options with bid quotes above the SA bounds, $C^i_{bt}(I_t, K_i, T) > \overline{C}_i(I_t, K_i, T)$, and a similar one for puts. (2) Short each overpriced option, allocate the proceeds to the index and the riskless asset as in (2.6)-(2.7), and close the position on the next trading day. We close the option position at the SA upper bound as in (3.2), at the equilibrium value of the option corresponding to the chosen RRA, or at the ask price of the option. The first two are proxies for the mispricing in the frictionless world and the latter for the mispricing in the economy with partial frictions. (3) Compute portfolio returns for OT.[15] (4) At each cross section with mispricing, the total return is the weighted average of individual option returns (2.8), with weights proportional to the differences of the bid prices from the option upper bounds. The positive weights sum to one in each cross section and maximize the total mispricing. (5) These portfolio returns are the excess OT returns, which are calculated across all cross sections including those with no overpriced options, which have identical IT and OT returns. (6) The time series of IT and OT returns are standardized to a common index value to meet the requirements of out-

[15] Given the independence of daily returns under our dynamics, it is enough to compute one-day returns for all cross sections for our tests. In more complex index dynamics, we may need to apply (1.8) for returns at option maturity.

of-sample tests in the following section, which assume that the IT and OT return series are drawn from the same respective distributions.

The first three rows in the top (bottom) panel of Table 4 show the realized excess return of the OT minus IT portfolio returns for calls (puts) in annualized format, its annual volatility, and the resulting information ratio. The last three rows in the top (bottom) panels show the number of overpriced contracts and percentage of cross sections with positive OT portfolio returns for calls (puts). The results in Panels A to E reflect two different RRA values for the IT-OT comparisons. In Panels A to C, the RRA φ^* of the representative IT investor is equal to 4 for all cross sections, significantly lower than the minimum upper bound-implied RRA across all cross sections. Panel A shows the test results for the frictionless market when the option positions are closed at the SA upper bound. Panel B and C are identical to A with respect to the portfolio strategies, except that the option positions are closed at the prevailing equilibrium option prices and at the ask prices of the options, respectively. In all panels, we report statistics when the option portfolio is set 9-, 7-, and 4-days-to-maturity.

As expected, the profits from the short option strategies are higher in Panel B, when the positions are closed at the equilibrium price, compared to Panel A, when the option position is closed at the upper bounds. These tests are resounding confirmations of the Jouini and Kallal (1995) theoretical result, that frictionless option prices that do not lie inside the bid-ask spread are mispriced. Panel C shows why these mispriced options persist, which is very simply the fact that they are not mispriced in the market with frictions, even the partial ones that we have included!

Looking across information ratios in Panels A and B of Tables 4 and OA6, we observe that the information ratio is the highest 9 days before maturity in the frictionless setting and it decreases gradually as we get closer and closer to the maturity. Since the SA bounds do not change very much because of the small differences in maturities, the change can be attributed to heavy trading in the mispriced options that came closer to the SA bounds. However, the excess returns and information ratio increase a day before maturity. The investigation of these trading patterns is left for further research.

[Table 4 about here]

Panels D and E of Table 4 show the exploitation of the mispriced ATM options by more risk averse CRRA investors, with an RRA $\hat{\varphi}$ higher than φ^*. These investors would need to modify the strategies on the basis of their risk preference multiplier introduced in the previous subsection. However, they still reap excess risk adjusted profits by exploiting option quotes violating the SA bounds. As outlined in the previous subsection, investors who are more risk averse than the filtered IT investor with RRA equal to φ^*, with

$\hat{\varphi} > \varphi_{j,t:T}^{\max}$ for some $j = 1,...,n$, must short $\zeta_j = \dfrac{\varphi_{t,T}^{\max}}{\hat{\varphi}} < 1$ units of overpriced option contracts, with the proceeds allocated as in (2.6)-(2.7).

We observe that OT excess returns are uniformly lower (first row) when the investors are more risk averse, as expected, than in the corresponding returns in Panels A-C of Table 4, but the standard deviations are also lower (second row), resulting in small changes in the information ratios. We conclude that, although mispriced options can be exploited in frictionless world for all investors regardless of their risk preferences, these mispricing do not survive in the presence of frictions.

Figure OA2 of the online appendix shows the time series of the percentages of overpriced ATM (.98<K/S<1.02) SPXW options in our sample for both calls and puts across each DTM group. It shows that between 60% and 100% of the ATM calls (left panel) and puts (right panel) are overpriced between 2011 and 2020, given the SA option bounds. This percentage, however, decreased sharply to around 20% during the last two Covid years, a decrease that could be related to an influx of individual traders into option markets.[16] We illustrate this observed mispricing in the frictionless world by showing in Figure 1 the cumulative OT portfolio returns from the mispriced portfolios separately for calls and puts in our sample. Comparing these cumulative OT returns with the contemporary cumulative IT returns in Figure OA3, it appears clearly that the enhancement from the overpriced options is significant, as confirmed by our statistical tests presented in the next subsection.

[Figure 1 about here]

As discussed before, a major advantage of SA is the identification of mispriced options in frictionless trading, among which are also the mispriced options in the market with frictions. We explore this hypothesis by setting zero-net-cost OT portfolios of index and options using all but the previously identified overpriced options. Figure OA3 in the online appendix shows the cumulative excess returns of these portfolios for options with 7 DTM. It is obvious that these portfolios offer no advantage to investors and are, if anything, decreasing the cumulative returns of the IT investor. We conclude that our SA bounds' violations are excellent identifiers of overpriced options in the frictionless world, among which overpricing under frictions may also be sought.

3.6 The ex-post verification of frictionless mispricing

Table 4 also presents results of the ex-post verification of frictionless mispricing of ATM end-of-week SPXW options. The top panel (bottom panel) reports *p*-values for the Davidson-Duclos (DD, 2013) tests,

[16] Such an influx was noted in a recent paper by Bryzgalova, Pavlova and Sikorskaya (2022) for equity options.

to verify whether the time series of OT call (put) portfolio returns across all cross sections (477 expiring Fridays) stochastically dominates the corresponding time series of IT returns in the second degree, separately for each maturity group. These tests were first used in Finance by Constantinides *et al* (2011) and are particularly convincing because the null is non-dominance of OT over IT, or $H_0 : OT \nsucc_2 IT$. Rejection of the null, therefore, implies that OT portfolios dominate IT portfolios in the second degree ($OT \succ_2 IT$) and thus every risk averse trader would choose the option portfolios in the frictionless world to increase her expected utility.

The middle three rows in each panel shows the *p*-values for the difference of OT and IT mean of portfolio returns for each right tail trimming. Consistent with the DD theory, the null hypothesis is tested under three different conditions with respect to handling the joint support of the paired sample. In all cases, there is a 10% trimming of the left tail of the distribution, while the right tail has a 0%, 5% and 10% trimming. The inference becomes progressively weaker as the right tail trimming increases.

The results of both call and put panels are clear and unequivocal for all three DTM groups and all three closing strategies, although there are some interesting and potentially informative differences between the DTM groups. Under frictionless market assumptions (Panels A, B and D), the DD tests reject the non-dominance null everywhere regardless of whether the option position is closed at the upper bound (Panel A) or at the equilibrium price with RRA equal to 4 (Panel B). In all these scenarios the *p*-values of the DD tests are equal to zero or very low everywhere, although, as expected, the profits from the short option strategies are higher in Panel B, when the positions are closed at the equilibrium price. The DD tests in Panel C are consistent with the non-dominance null in the 9- and 4-day DTM groups, since the excess OT returns are negative. Table OA6 of online appendix shows that the 8-day DTM group is very similar to the 9-day group, and the frictionless DD tests also reject the non-dominance null for all other DTM groups.

The 7-day maturity is somewhat different in the market with friction (Panel C) as the DD test implies that the excess OT return is positive (but small) and significant at 5% and 10% right trimming, although not at the no trimming. Nonetheless, it is similar in its frictionless metrics to the 9-day maturity for both calls and puts, even though that maturity does not survive as profitable trading in the market with frictions. Panel D (E) of Table 4 shows the results of the DD test for a more risk averse IT investor with an RRA equal to $\hat{\varphi} = 14.16$ in the frictionless world (in the presence of partial frictions). We note that the DD test results are quite similar to those in Panels A (C). Thus, statistical inference confirms our previous statements that mispriced options can be exploited in the frictionless world by all risk averse investors regardless of their risk preferences, but such mispricing does not generally survive in the presence of frictions; see also note 9.

The results of Table 4 also illustrate the ranges of RRA parameter consistent with a frictionless equilibrium within the SA bounds for each DTM group. An obvious exercise for anyone who wishes to extract the best-fitting frictionless equilibrium based on the "correctly" priced deep OTM puts would be the extraction of a common RRA parameter from the bid-ask spread midpoints that happen to be within the SA bounds in all cross sections. This can then be extended to all degrees of moneyness beyond the OTM puts, which will not, in general correspond to an observed option market value. Figure OA1 shows the distribution of the SA implied upper bounds on RRA for different maturity categories. Further analysis of implied RRA is left for future research.

3.7 Robustness checks

We close our empirical work on SPXW options by verifying mispricing in the market with frictions with the few empirical tools available as of this writing. The latest one is the CCP (2020) algorithm, which was applied to a time series of 257 SPXW options extending from January 2006 to February 2013 for DTM=7. They identified 227 mispriced cross sections showing an annualized excess return of 1.94%, with the DD tests decisively rejecting the non-dominance null. Nonetheless, the mispricing tests were on empirically selected portfolios of call and put options of all degrees of moneyness. As such, they do not provide new insights to our more detailed results and are left for future research, with extended modelling of the market with frictions.

More relevant for detecting mispricing in the presence of frictions is a filter for overpriced call options in the form of the CP (2002) upper bound, in which a short call is added to the index holdings of the IT investor. The resulting upper bound is equal to the *P*-expectation of the payoff discounted by the cum dividend expected return of the index and multiplied by the roundtrip transaction costs $\frac{1+k}{1-k}$, where k is the transaction cost rate on the index; since it is of the order of 0.1 basis points, it is omitted. We find violating call bid prices in about 438 cross sections depending on the assumed risk premium, which yield an excess OT return of 0.34%. The enhancement is rather weak, but it does survive the DD rejection of the non-dominance null tests. We conclude that the SPXW call options are also mispriced in the economy with frictions. Full tests for possible mispricing in the economy with frictions must consider the nonexistence of a monotone kernel passing through the cross sections' bid-ask spreads. As noted in the introduction such nonexistence is necessary and sufficient for profitable trading in zero net cost option portfolios. Given the segmentation of the market for calls and puts, the nonexistence is to be expected in most cross sections.

Last, we examine whether our key empirical result, the systematic underpricing by the SA upper bound of entire regions of highly liquid moneyness zones for both call and put options compared to the observed option market data, may be due to our estimation of the asset dynamics. It is easy to see from (2.3) and (2.5) that a simple way of raising the SA upper bounds towards the observed market prices is the increase in the left tail truncation $j > j_{\min} \equiv \underline{j}$ of the jump amplitude. This may raise the *P*-volatility in (2.3), but it also raises the *Q*-volatility's risk premium at the SA upper bound for each cross section, depending also on the corresponding riskless rate *r*. As it turns out, however, such a raise would worsen significantly the fit of the model dynamics to the observed data. Even with the estimated model parameters, for all DTM the average predicted annualized volatility over all cross sections exceeds the average observed ex post volatility of the observed returns by a non-negligible amount, equal to 0.272% for the 7-day maturity. We conclude that the observed overpricing in the option market, compared to any frictionless equilibrium consistent with the index return data, is real and not due to our estimation methodology.

4. Conclusions

Our study provides evidence that extracting the frictionless *Q*-dynamics from the observed bid-ask midpoint in the short-term index option market is a futile effort on both theoretical and empirical grounds. The observed option market data is fundamentally inconsistent with the frictionless option price format as implied by the underlying index returns. There exists a clear segmentation of the markets for SPXW call and put options and strong moneyness effects in these options. More importantly, for the highly liquid ATM calls/puts that represent more than 70% of the SPXW trading volume, not only the bid-ask midpoint but also the entire width of the quotes lay above their SA option bounds in most of the cross sections. These non-overlapping (mispriced) options were used to generate zero-net-cost portfolio returns that are statistically and economically significant in the frictionless world. However, such portfolio returns disappear when trading options at the appropriate bid and ask prices. This frictionless mispricing of the SPXW options is consistent with the documented mispricing in the few studies that recognized frictions and with the even fewer theoretical studies that compared option pricing without and with frictions.

Our SA approach also provides some insight about the estimation of left tail risk in portfolios closely correlated with the S&P 500 index. Empirical studies typically use Value at Risk (VaR), expected shortfall and higher order return moments, and evaluate it by using specific forms of the utility function, asfor instance Bali *et al* (2009), and Atilgan *et al* (2019). Our own results show that the deep OTM SPXW put options are "correctly" priced. If this also holds for longer maturities, then long positions in such options offer protection against left tail risk. The values of these options are obvious estimates of such risk.

We also note that frictionless equilibrium in the option market as modelled in various NAE studies often yielded less than fully satisfactory results in modelling left tail risk. Recall the AFT comments about the sharp separation between the dynamics of jump risk and its pricing (p. 1370), and the similar failure of Ross' (2015, pp. 642-643) Recovery Theorem, in which the risk neutral left tail predicted an unreasonably higher probability of very large drops in the index than was ever observed in practice. By contrast, our empirical work shows that the market for SPXW options prices efficiently left tail risk, insofar as in a large majority of the cross sections there exist frictionless equilibria consistent with the observed prices of deep OTM put options, and there do not exist zero net cost option positions profitable for *all* IT investors.[17] While under SA there does not exist a unique frictionless equilibrium for all cross sections, the assumption of a CRRA investor as an IT-OT trader allows the modelling of a frictionless equilibrium via (3.1)-(3.2) at the level of individual options. This sets upper limits on the RRA of a "representative" investor for the equilibrium in each cross section, and even tighter limits on the RRA of investors that can represent the equilibria of all cross sections in the data. In all cases, it is the pricing of jump risk that defines the appropriate RRA limits.

For IT traders more risk averse than such RRA levels, their optimal allocations place them outside the SA bounds in the corresponding cross sections. In spite of this, there were highly profitable short option portfolio strategies for both calls and puts tailored to each RRA level, shown in Panels (A, B) and (D, E) for RRA levels of 4 and 14.16, respectively, that were independent of whatever equilibrium could be extracted from a representative investor satisfying (3.1). We conclude that the weaker assumptions of the SA approach provide insights about frictionless equilibrium in the SPXW market and the implied short-term risks that are not feasible with the NAE models based on observed option market data.

The inconsistency of the dynamics of the index with the risk neutral distribution extracted from option market data is also found in longer maturity SPX options that adopt the frictionless NAE approach based on observed option market data. It has given rise to a long debate about the shape of the pricing kernel and the alleged overpricing of OTM put options.[18] In order to verify whether such an inconsistency is also due to the disconnect between the option market data and the SA bounds we need methodological extensions of the derivation of these bounds to index dynamics that allow the stochastic evolution of volatility during the maturity of the options, which is left for future studies.

Our results for the resolution of these issues suggest that a starting point should be the recognition that there is an intermediate market where the bid and ask option prices are set. To our knowledge, the only study that paid at least lip service to the intermediate option market is Garleanu, Pedersen and Poteshman (2009). However, it assumes a free entry perfectly competitive frictionless market, in which there is no information

[17] Recall that there are several studies claiming that index put options are too expensive, especially the deep OTM ones; see, for instance, Bondarenko (2014).
[18] See, for instance, Jackwerth (2000) for the shape of the kernel and Driessen and Maenhout (2007) for the overpriced put. The long list of references on the debate, which was still ongoing as of 2020, is surveyed in Perrakis (2022).

asymmetry and no bid-ask spreads or any other types of frictions. Further, their traders belong to the Constant Absolute Risk Aversion (CARA) class of utility functions, which are inconsistent with most of the experimental and empirical evidence that finds decreasing absolute risk aversion.[19] Their market structure assumptions are also fundamentally inconsistent with the SPXW option market.

To begin with, there are wealth-based economies of scale in quoting options, shown in Perrakis (2019, pp. 233-239) and repeated in our online appendix, which may keep retail traders away from the option market. It is also known that there is exactly one designated liquidity provider for the entire SPXW class, known as DPM; see https://www.cboe.com/us/options/trading/liquidity_providers/. The Cboe Rule book, available at https://cdn.cboe.com/resources/regulation/rule_book/cboe-exchange-inc-rule-book.pdf, indicates clearly that the DPM is in charge of the order book (p. 329) during regular trading hours, and is entitled to priority when facing competition in executing trades. This entitlement guarantees at least 30% of the trade volume in the order book (p. 252), and together with information asymmetry and economies of scale it constitutes a powerful barrier to entry. This is also consistent with data that we present from the Cboe in Table OA5 of our online appendix, showing that the trading volume by market makers is between 44% and 54% on average for all maturities and degrees of moneyness of SPXW options.

In our online appendix we present an equilibrium model of a competitive market in the presence of frictions in trading index futures for traders that belong to the CRRA class of investors, in which we show that the option bid and ask quotes are, indeed, consistent with the frictionless SA bounds, as in Jouini and Kallal (1995). We also formulate models that take into account the monopolistic liquidity provider, who sets the size and depth of the quotes and hedges her positions in the presence of transaction costs while facing entry from other large firms that compete for both long and short options in a Cournot-style oligopoly . These models can be taken only as templates for empirical work, which is a major undertaking and transcends the scope of this paper.

References

Amin, K. I., and V. K. Ng, 1993, “Option Valuation with Systematic Stochastic Volatility,” *Journal of Finance* 48, 881-910.

Andersen, T. G., N. Fusari, and V. Todorov 2017, “Short-term market risks implied by weekly options,” *The Journal of Finance* 72 (3), 1335–1386.

Arvanitis, S., T. Post and N. Topaloglou, 2021, “Stochastic Bounds for Reference Sets in Portfolio Analysis”, *Management Science* 67, 7737-7754.

[19] See Friend and Blume (1975). Using CARA utilities also obfuscates the economies of scale in option quotes due to wealth; see Perrakis (2019, p. 237).

Atilgan, Y., T. G. Bali, K. O. Demirtas, and A. D. Gunaydin, 2020, “Left Tail Momentum: Underreaction to Bad News, Costly Arbitrage and Equity Returns”, *Journal of Financial Economics* 135, 725-753.

Bali, T. G., K. O. Demirtas, and H. Levy, 2009, “Is There an Intertemporal Relation Between Downside Risk and Expected Returns?”, *Journal of Financial and Quantitative Analysis* 44, 883-909.

Barone-Adesi, G., N. Fusari, A. Mira, and C. Sala, 2020, “Option Market Trading Activity and the Estimation of the Pricing Kernel: a Bayesian Approach”, *Journal of Econometrics* 216, 430-449.

Bates, D. S., 1991, “The Crash of ’87: Was it Expected? The Evidence from Option Markets,” *Journal of Finance* 46, 1009-1044.

Beare, B. K., 2011, “Measure Preserving Derivatives and the Pricing Kernel Puzzle,” *Journal of Mathematical Economics* 47, 689-697.

Beare, B. K., 2023, “Optimal Measure Preserving Derivatives Revisited”, forthcoming in *Mathematical Finance*.

Bogle, John C., 2005, “The Mutual Fund Industry 60 Years Later: For Better or Worse?”, *Financial Analyst Journal* 61, 15–24.

Bryzgalova, S., A. Pavlova, and T. Sikorskaya, 2022, “Retail Trading in Options and the Rise of the Big Three Wholesalers”, *working paper,* London Business School.

Charles D. E., 2017, “The End of Active Investing? Technology and Low Returns Have Delivered a Killer Blow to a Once Dominant Industry”, *Financial Times*, available at https://www.ft.com/content/6b2d5490-d9bb-11e6- 944b-e7eb37a6aa8e.

Chatrath. A., R. A. Christie-David, H. Miao, S. Ramchander, 2015, “Short-term Options: Clienteles, Market Segmentation, and Event Trading”, *Journal of Banking and Finance* 61, 237-250.

Chatrath. A., R. A. Christie-David, H. Miao, S. Ramchander, 2019, “Losers and Prospectors in the Short-term Options Market”, *Journal of Futures Markets* 39, 721-743.

Constantinides, G. M., M. Czerwonko, J. C. Jackwerth, and S. Perrakis, 2011, “Are Options on Index Futures Profitable for Risk Averse Investors? Empirical Evidence,” *Journal of Finance* 66, 1407-1437.

Constantinides, G. M., M. Czerwonko, and S. Perrakis, 2019, “Mispriced index option portfolios,” *Financial Management*, 49, 2, 297-330.

Constantinides, G. M., J. C. Jackwerth, and S. Perrakis, 2009, “Mispricing of S&P 500 Index Options,” *Review of Financial Studies*, 22, 1247-1277.

Constantinides, G. M. and L. Lian, 2019, “The supply and demand of S&P 500 put options,” *Critical Finance Review* 10, 1-20.

Davidson, Russell, and Jean-Yves Duclos, 2013, “Testing for Restricted Stochastic Dominance,” *Econometric Reviews* 32: 84–125.

Driessen, Joost, and Pascal J. Maenhout, 2007, “An Empirical Portfolio Perspective on Option Pricing Anomalies,” *Review of Finance* 11, 561-603.

Fournier, M., and K. Jacobs, 2020, “A Tractable Framework for Option Pricing with Dynamic Market Maker Inventory and Wealth”, *Journal of Financial and Quantitative Analysis* 55, 1117-1162.

Friend, Irwin., and M. E. Blume, 1975, “The Demand for Risky Assets”, *American Economic Review* 65, 900-922.

Fu, M. C., B. Li, G. Li, and R. Wu, 2018, “option Pricing for a Jump Diffusion Model with General Discrete Jump Size Distributions”, *Management Science* 63, 3961-3977.

Garleanu, Nicolae, Lasse H. Pedersen, and Allen M. Poteshman, 2009, “Demand Based Option Pricing,” *Review of Financial Studies* 22, 4259-4299.

Ghanbari, H., M. Oancea and S. Perrakis, 2021, “Shedding Light in a Dark Matter: Jump-Diffusion and Option-Implied Investor Preferences”, *European Financial Management* 27, 244-286.

Heston, S. L., 1993, “A Closed-Form Solution for Options with Stochastic Volatility, with Applications to Bond and Currency Options.” *Review of Financial Studies,* 6, 327-344.

Heston, S. L., and S. Nandi, 2000, “A Closed-form GARCH Option Valuation Model.” *Review of Financial Studies*, 13, 585-625.

Jackwerth, J., 2000, “Recovering Risk Aversion from Option Prices and Realized Returns,” *Review of Financial Studies* 13, 433-451.

Jouini, E., and H. Kallal, 1995, “Martingales and Arbitrage in Securities Markets with Transaction Costs”, *Journal of Economic Theory* 66, 178-197.

Kou, SG., 2002, “A Jump-Diffusion Model for Option Pricing”, *Management Science* 48, 1086-1101.

Kou, SG., and H. Wang, 2004, “Option Pricing Under a Double Exponential Jump-Diffusion Model”, *Management Science* 50, 1178-1192.

Levy, H., 1985, “Upper and Lower Bounds of Put and Call Option Value: Stochastic Dominance Approach,” *Journal of Finance* 40, 1197-1217.

Merton, R. C., 1976, “Option Pricing when the Underlying Stock Returns are Discontinuous,” *Journal of Financial Economics* 3, 125-144.

Perrakis, S., 1986. “Option Bounds in Discrete Time: Extensions and the Pricing of the American Put.” *Journal of Business* 59, 119-141.

Perrakis, S., 2019, “*Stochastic Dominance Option Pricing: An Alternative Approach to Option Market Research*.” Palgrave Macmillan.

Perrakis, S., 2022, “From Innovation to Obfuscation: Continuous Time Finance Fifty Years Later”, *Financial Markets and Portfolio Management* 36, 369-401.

Ghanbari, H, and I. M. Oancea, and S. Perrakis, 2022, “Stochastic Dominance, Stochastic Volatility and the Prices of Volatility and Jump Risk” available at SSRN: https://ssrn.com/abstract=3999387

Perrakis, S. and P. J. Ryan, 1984. “Option Pricing Bounds in Discrete Time.” *Journal of Finance* 39, 519-525.

Post, Thierry, and I. Longarela, 2021, “Stochastic Arbitrage Opportunities for Stock Index Options”, *Operations Research* 69, 100-113.

Ritchken, P. H., 1985, “On Option Pricing Bounds,” *Journal of Finance* 40, 1219-1233.

Ritchken, P. H., and S. Kuo, 1988, “Option Bounds with Finite Revision Opportunities,” *Journal of Finance* 43, 301-308.

Ross, S. 2015, The Recovery Theorem. *Journal of Finance* 70, 615–648.

Shleifer, Andrei and Robert W. Vishny, 1997, The limits of arbitrage, *The Journal of Finance* 52: 1 (March), 35-55.

Wiggins, J. 1987, “Option Values Under Stochastic Volatility: Theory and Empirical Estimates.” *Journal of Financial Economics*, 5, 351-372.

Table 1: Summary Statistics Weeklys - 4 Days-to-Maturity (Monday)

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	1,081	863	2,474	3,437	4,309	3,761	2,198	1,017	919	20,059
% Contracts	5.4%	4.3%	12.3%	17.1%	21.5%	18.7%	11.0%	5.1%	4.6%	100%
Avg IV	71.8%	34.2%	25.0%	19.1%	15.6%	13.8%	16.1%	21.0%	36.0%	21.2%
Avg Mid-Quotes	622.7	255.6	157.2	89.4	40.0	10.5	3.6	2.5	2.4	90.5
Relative BA Spread	1.4%	2.0%	2.9%	3.7%	3.3%	5.8%	22.4%	39.9%	56.0%	10.0%
Option Bounds Spread	0.1%	0.5%	1.0%	1.9%	4.9%	18.2%	29.7%	30.8%	23.1%	10.8%
Avg NetBuy MM	0	2	-5	-18	4	23	8	-26	-22	0.2
% Volume MM	54%	56%	49%	47%	52%	51%	49%	47%	50%	51%
Total Volume	12	13	103	269	2,400	7,351	2,177	666	337	13,327
Total Open Interest	93	208	1,493	3,932	11,599	14,696	4,974	1,657	1,072	39,723
Panel B: Calls Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	313	529	1,487	1,731	642	279	216	101	47	5,345
	29%	61%	60%	50%	15%	7%	10%	10%	5%	27%
Overpriced Contracts	37	54	234	776	2,470	1,873	426	163	301	6,334
	3%	6%	9%	23%	57%	50%	19%	16%	33%	32%
Cross-sections >1 Overpriced	11	14	53	153	335	307	88	22	14	345
	4%	5%	13%	36%	78%	72%	33%	18%	22%	81%
Cross-sections >50% Overpriced	6	14	35	90	272	261	66	19	11	152
	2%	5%	9%	21%	64%	61%	25%	15%	17%	36%
Cross-sections 100% Overpriced	3	14	21	72	158	230	61	16	11	4
	1%	5%	5%	17%	37%	54%	23%	13%	17%	1%
Panel C: Summary Statistics Puts										
# Contracts	1,123	4,590	6,405	4,319	4,340	3,991	1,941	891	1,472	29,072
% Contracts	3.9%	15.8%	22.0%	14.9%	14.9%	13.7%	6.7%	3.1%	5.1%	100%
Avg IV	44.7%	31.4%	24.3%	19.1%	15.5%	13.4%	17.6%	23.8%	49.5%	23.1%
Avg Mid-Quotes	2.3	1.2	2.0	4.7	12.1	37.6	86.3	144.2	402.0	38.9
Relative BA Spread	20.5%	35.6%	23.2%	10.3%	4.3%	4.9%	4.7%	3.7%	2.6%	14.9%
Option Bounds Spread	155.8%	147.2%	127.9%	94.8%	41.8%	2.5%	0.4%	0.2%	0.0%	78.1%
Avg NetBuy MM	3	-44	11	65	2	-4	-6	-8	-6	4
% Volume MM	50%	44%	48%	48%	52%	52%	50%	50%	50%	49%
Total Volume	387	2,338	6,878	7,737	8,007	1,973	253	81	70	27,724
Total Open Interest	1,575	12,456	22,548	17,188	14,504	5,505	1,913	945	1,291	77,925
Panel D: Puts Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	801	3,079	4,123	2,388	706	339	210	91	56	11,793
	71%	67%	64%	55%	16%	8%	11%	10%	4%	41%
Overpriced Contracts	296	627	1,197	1,582	2,763	1,502	193	67	117	8,344
	26%	14%	19%	37%	64%	38%	10%	8%	8%	29%
Cross-sections >1 Overpriced	24	52	105	231	350	292	35	11	5	365
	19%	14%	25%	54%	82%	68%	9%	5%	4%	85%
Cross-sections >50% Overpriced	23	46	74	155	310	168	17	8	1	66
	18%	12%	17%	36%	72%	39%	5%	4%	1%	15%
Cross-sections 100% Overpriced	21	43	57	104	204	51	12	5	1	1
	17%	11%	13%	24%	48%	12%	3%	2%	1%	0%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed on Monday at 3:00 PM from 20110317 to 20220317 with one week to maturity for the total of 428 expiring Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. "Relative BA Spread" ("Option Bounds Spread") is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). "Avg NB MM" is the average number of contracts purchased minus sold by market makers. "Total Volume" and "Total Open Interest" are in thousands. "Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table 2: Summary Statistics Weeklys - 7 Days-to-Maturity (Friday)

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	1,204	965	2,681	3,667	4,594	4,095	2,471	1,286	1,327	22,290
% Contracts	5.4%	4.3%	12.0%	16.5%	20.6%	18.4%	11.1%	5.8%	6.0%	100%
Avg IV	64.9%	33.9%	25.4%	19.8%	16.3%	14.3%	16.1%	19.8%	31.0%	21.4%
Avg Mid-Quotes	614.4	249.1	155.0	90.7	42.9	13.0	4.7	2.8	2.4	89.6
Relative BA Spread	1.6%	2.2%	3.2%	3.5%	3.1%	5.3%	16.5%	34.8%	64.4%	10.3%
Option Bounds Spread	0.2%	0.7%	1.2%	2.4%	6.0%	20.6%	32.3%	33.0%	25.9%	12.6%
Avg NetBuy MM	2	3	-8	-10	14	37	112	87	8	25
% Volume MM	60%	52%	52%	51%	51%	50%	50%	46%	50%	50%
Total Volume	16	17	107	259	2,562	7,871	2,790	926	509	15,058
Total Open Interest	89	254	1,382	3,953	11,787	13,818	4,976	1,996	1,477	39,732
Panel B: Calls Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	405	622	1,514	1,438	621	248	358	137	87	5,430
	34%	64%	56%	39%	14%	6%	14%	11%	7%	24%
Overpriced Contracts	49	104	483	1,411	3,202	2,526	634	172	183	8,764
	4%	11%	18%	38%	70%	62%	26%	13%	14%	39%
Cross-sections >1 Overpriced	15	28	106	255	404	373	136	27	13	415
	5%	9%	24%	55%	88%	81%	46%	18%	16%	90%
Cross-sections >50% Overpriced	9	25	70	177	360	338	115	21	8	217
	3%	8%	16%	38%	78%	73%	39%	14%	10%	47%
Cross-sections 100% Overpriced	6	24	54	122	249	298	95	19	7	5
	2%	8%	12%	27%	54%	65%	32%	13%	9%	1%
Panel C: Summary Statistics Puts										
# Contracts	2,343	5,944	6,952	4,652	4,640	4,311	2,176	976	1,467	33,461
% Contracts	7.0%	17.8%	20.8%	13.9%	13.9%	12.9%	6.5%	2.9%	4.4%	100%
Avg IV	43.0%	31.1%	24.8%	19.9%	16.3%	13.9%	16.9%	22.5%	39.0%	23.9%
Avg Mid-Quotes	2.7	1.8	3.2	7.0	15.3	39.8	86.1	145.1	350.4	34.6
Relative BA Spread	20.6%	24.0%	14.7%	7.3%	3.9%	4.6%	4.8%	3.8%	2.9%	11.4%
Option Bounds Spread	157.6%	150.1%	128.7%	96.0%	44.5%	3.2%	0.5%	0.2%	0.1%	84.4%
Avg NetBuy MM	35	-21	96	84	72	44	8	3	-10	46
% Volume MM	48%	43%	47%	49%	50%	50%	50%	50%	49%	48%
Total Volume	1,058	5,209	8,337	8,496	8,691	2,143	215	80	87	34,315
Total Open Interest	2,567	14,151	20,632	15,362	13,257	5,083	1,775	799	1,043	74,668
Panel D: Puts Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	1,637	4,387	4,642	2,104	699	274	233	90	56	14,122
	70%	74%	67%	45%	15%	6%	11%	9%	4%	42%
Overpriced Contracts	608	1,169	1,997	2,392	3,373	2,194	270	55	38	12,096
	26%	20%	29%	51%	73%	51%	12%	6%	3%	36%
Cross-sections >1 Overpriced	46	94	178	312	406	371	57	12	4	418
	20%	21%	39%	68%	88%	81%	14%	5%	3%	91%
Cross-sections >50% Overpriced	44	83	123	248	377	274	24	5	1	125
	19%	19%	27%	54%	82%	60%	6%	2%	1%	27%
Cross-sections 100% Overpriced	40	72	105	181	287	93	16	4	1	0
	17%	16%	23%	39%	62%	20%	4%	2%	1%	0%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed on Friday at 3:00 PM from 20110317 to 20220317 with one week to maturity for the total of 461 expiring Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. “Relative BA Spread” (“Option Bounds Spread”) is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). “Avg NB MM” is the average number of contracts purchased minus sold by market makers. “Total Volume,”“Avg Volume,” and “Total Open Interest” are in thousands. “Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table 3: Summary Statistics Weeklys - 9 Days-to-Maturity (Wednesday)

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	776	688	2,047	3,182	4,407	4,062	2,749	1,459	1,394	20,764
% Contracts	3.7%	3.3%	9.9%	15.3%	21.2%	19.6%	13.2%	7.0%	6.7%	100%
Avg IV	53.5%	30.8%	24.2%	18.8%	15.6%	13.6%	14.5%	17.5%	27.7%	19.0%
Avg Mid-Quotes	560.7	251.7	165.3	95.9	47.2	15.9	5.5	3.3	3.0	74.6
Relative BA Spread	1.6%	2.1%	2.6%	2.7%	2.4%	3.9%	11.0%	21.3%	38.9%	7.6%
Option Bounds Spread	0.2%	0.9%	1.7%	3.2%	7.5%	22.2%	33.3%	35.6%	31.4%	15.7%
Avg NetBuy MM	0	1	-6	-7	-3	7	24	-8	16	3
% Volume MM	53%	50%	51%	54%	51%	50%	50%	50%	51%	50%
Total Volume	9	13	55	133	1,565	4,070	1,515	361	249	7,970
Total Open Interest	81	194	1,105	3,410	10,283	13,045	5,491	2,185	1,851	37,646
Panel B: Calls Mispricing vs. SD Bounds										
Mid-Quotes Within Bounds	222	396	1,004	1,112	623	243	361	236	213	4,410
	29%	58%	49%	35%	14%	6%	13%	16%	15%	21%
Overpriced Contracts	77	125	624	1,529	3,140	2,411	745	147	112	8,910
	10%	18%	30%	48%	71%	59%	27%	10%	8%	43%
Cross-sections >1 Overpriced	19	35	121	278	369	334	162	24	13	392
	7%	14%	32%	66%	87%	79%	52%	14%	13%	92%
Cross-sections >50% Overpriced	10	29	81	187	343	311	137	19	7	242
	4%	12%	21%	44%	81%	73%	44%	11%	7%	57%
Cross-sections 100% Overpriced	4	26	57	142	253	277	112	17	4	7
	2%	11%	15%	34%	60%	65%	36%	10%	4%	2%
Panel C: Summary Statistics Puts										
# Contracts	3,479	5,874	6,613	4,502	4,489	3,995	1,745	731	1,027	32,455
% Contracts	10.7%	18.1%	20.4%	13.9%	13.8%	12.3%	5.4%	2.3%	3.2%	100%
Avg IV	37.1%	28.0%	22.7%	18.5%	15.5%	13.3%	15.7%	19.8%	37.6%	22.4%
Avg Mid-Quotes	2.5	2.2	4.4	9.0	18.9	43.9	90.9	147.2	358.0	30.4
Relative BA Spread	17.3%	18.0%	10.2%	5.5%	3.0%	3.5%	4.1%	3.3%	2.6%	9.2%
Option Bounds Spread	157.8%	148.1%	123.8%	89.1%	41.7%	4.1%	0.8%	0.3%	0.1%	87.6%
Avg NetBuy MM	-1	-8	-9	10	10	16	-12	15	-6	1
% Volume MM	44%	46%	48%	50%	51%	51%	49%	46%	45%	49%
Total Volume	855	1,829	3,268	3,211	3,749	974	134	62	61	14,142
Total Open Interest	5,882	13,732	17,906	12,975	10,584	4,123	1,510	666	862	68,240
Panel D: Puts Mispricing vs. SD Bounds										
Mid-Quotes Within Bounds	2,559	4,655	4,187	1,841	654	272	235	94	82	14,579
	74%	79%	63%	41%	15%	7%	13%	13%	8%	45%
Overpriced Contracts	878	1,072	2,277	2,518	3,271	2,075	205	49	32	12,377
	25%	18%	34%	56%	73%	52%	12%	7%	3%	38%
Cross-sections >1 Overpriced	54	86	191	313	368	330	51	7	4	396
	15%	20%	45%	74%	87%	78%	14%	3%	3%	93%
Cross-sections >50% Overpriced	49	64	132	223	344	280	19	5	0	109
	13%	15%	31%	53%	81%	66%	5%	2%	0%	26%
Cross-sections 100% Overpriced	48	55	88	173	277	104	14	4	0	1
	13%	13%	21%	41%	65%	25%	4%	2%	0%	0%

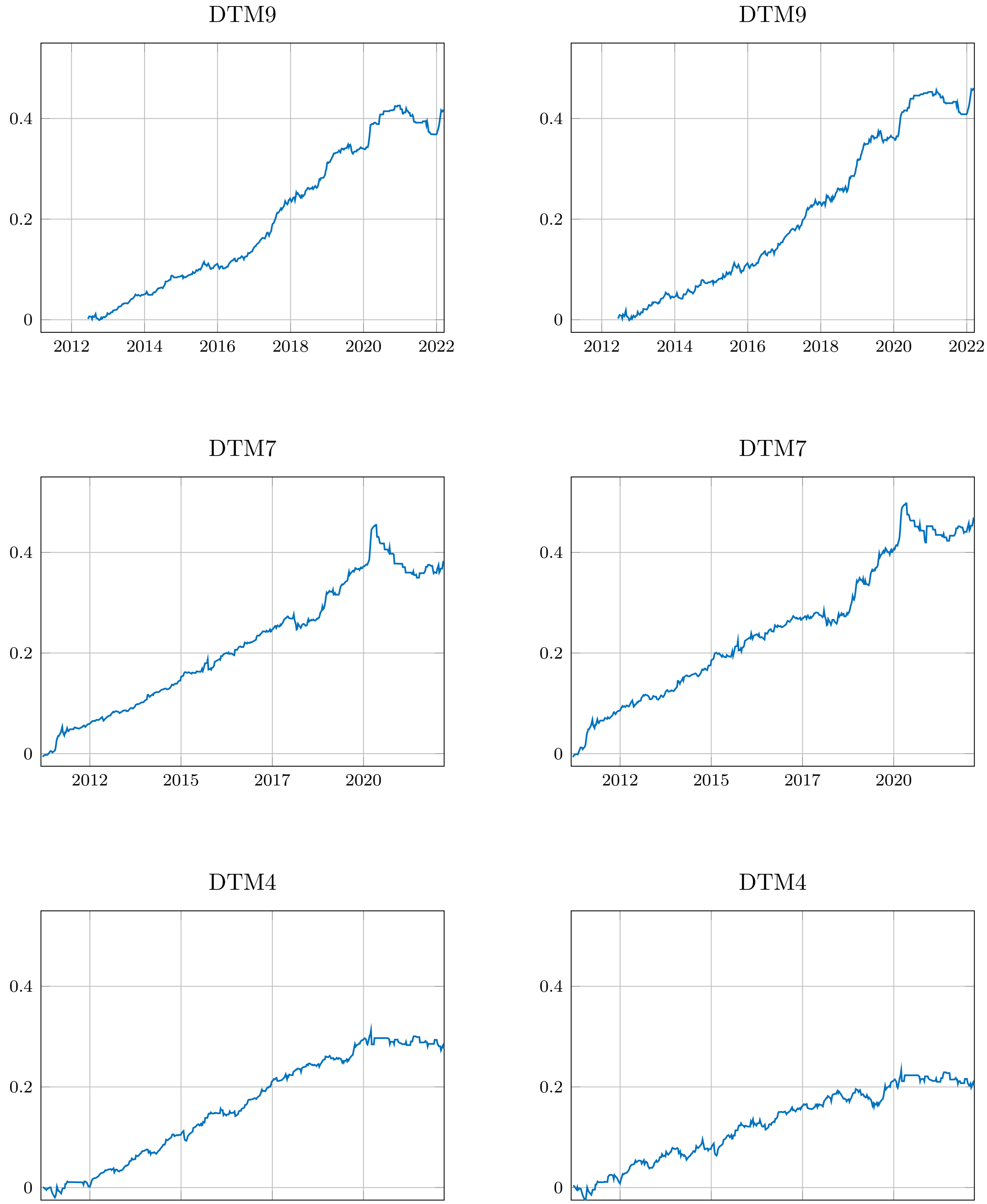
This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed on Wednesday at 3:00 PM from 20110317 to 20220317 with one week to maturity for the total of 424 expiring Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. "Relative BA Spread" ("Option Bounds Spread") is the average bid-ask (upper and lower bound) spreads in proportion of mid-quotes (mid-bounds). "Avg NB MM" is the average number of contracts purchased minus sold by market makers. "Total Volume" and "Total Open Interest" are in thousands. "Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table 4: Return Characteristics Option Trader Portfolios

	Panel A: Open @Bid - Close @UB			Panel B: Open @Bid - Close @NAE			Panel C: Open @Bid - Close @Ask			Panel D: Open @Bid - Close @UB			Panel E: Open @Bid - Close @Ask		
	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9
Calls:															
Avg Return	17.2%	23.7%	25.9%	17.7%	24.9%	29.3%	-10.0%	4.1%	-7.5%	13.6%	19.2%	20.5%	-8.5%	3.4%	-6.5%
St. Dev.	5.9%	6.8%	5.2%	5.9%	6.8%	5.2%	6.0%	6.4%	4.9%	5.0%	5.7%	4.3%	5.1%	5.4%	4.1%
Info Ratio	2.91	3.47	4.95	2.99	3.65	5.67	-1.66	0.64	-1.54	2.74	3.35	4.74	-1.65	0.63	-1.61
H_0: OT $\not\succ_2$ IT	0.001	0	0	0	0	0	1	0.206	1.000	0.002	0	0	1	0.210	1
H_0: OT $\not\succ_2$ IT (5% Trim)	0	0	0	0	0	0	1	0.011	1	0	0	0	1	0.009	1
H_0: OT $\not\succ_2$ IT (10% Trim)	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1
% CS Ret>0	52%	60%	60%	53%	60%	62%	39%	50%	42%	52%	60%	60%	39%	50%	42%
No. CS	318	361	354	318	361	354	318	361	354	318	361	354	318	361	354
No. Contracts	3,698	4,944	5,170	3,698	4,944	5,170	3,698	4,944	5,170	3,698	4,944	5,170	3,698	4,944	5,170
Puts:															
Avg Return	12.7%	29.1%	28.6%	18.6%	37.7%	41.7%	-12.3%	6.1%	-7.6%	9.9%	23.6%	22.6%	-10.4%	5.1%	-6.7%
St. Dev.	7.2%	8.4%	6.2%	7.2%	8.5%	6.2%	6.7%	7.4%	5.2%	6.0%	7.0%	5.1%	5.6%	6.2%	4.3%
Info Ratio	1.76	3.45	4.62	2.57	4.45	6.75	-1.84	0.82	-1.45	1.66	3.38	4.44	-1.87	0.82	-1.55
H_0: OT $\not\succ_2$ IT	0.013	0	0	0.003	0	0	1	0.15	1	0.021	0	0	1	0.144	1
H_0: OT $\not\succ_2$ IT (5% Trim)	0.005	0	0	0	0	0	1	0.014	1	0.006	0	0	1	0.004	1
H_0: OT $\not\succ_2$ IT (10% Trim)	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1
% CS Ret>0	47%	54%	55%	49%	56%	60%	40%	48%	42%	47%	54%	55%	40%	48%	42%
No. CS	332	362	353	332	362	353	332	362	353	332	362	353	332	362	353
No. Contracts	3,967	4,925	5,147	3,967	4,925	5,147	3,967	4,925	5,147	3,967	4,925	5,147	3,967	4,925	5,147

The table reports statistical properties of excess returns of option trader (OT) portfolio, including arithmetic average daily returns (annualized) and standard deviations. The row "% CS Ret>0" shows the percentage of weeks (trades) with positive portfolio returns. The returns in Panels A to C are computed when an OT writes an overpriced option at its bid quote and closes her position, respectively at the option upper bound, equilibrium price, and at the ask quote. Panels D and E reports similar results when OT is more risk averse. The trading strategy implement using 2% ATM options, end-of-week expiration, with non-zero volume in two consecutive days. Column DTM7 shows returns when the portfolio is set using options with four days-to-maturity on Monday and the positions are closed the next trading day. The OT portfolios are across all weeks in the sample and when there is no overpriced ATM option, OT=IT. The table also reports p-values for Davidson-Duclos (2007) second order stochastic dominance test for paired (correlated) outcomes.The row H_0: OT $\not\succ_2$ reports p-values for the null of non-dominance of time series of OT portfolio returns over time series of IT portfolio returns, with no trimming in the right tail (fourth row), 5% trimming in the right tail (fifth row), and 10% trimming in the right tail (six row).

Figure 1: Cumulative Excess Return OT Portfolio



This figure shows the cumulative return OT portfolio in excess of the index returns separately for calls (left panel) and puts (right panel) across all cross-sections. OT portfolio is set only with 2% ATM Weeklys, end-of-week expiration, with non-zero volume on any two consecutive trading days.

Stochastic Arbitrage and the Valuation of Weekly Options: Theory and Empirical Evidence

HAMED GHANBARI AND STYLIANOS PERRAKIS

Online Appendix

A. Proof of Equation 3.1

The kernel $\frac{I_T^{-\varphi}}{E_t[I_T^{-\varphi}]}$ is obviously monotone decreasing in I_T, from which it follows that the implied upper boundary *Q*-distribution is given by (2.4). Since the return distribution is independent and identically distributed according to (2.2) for every time partition Δt, we apply (2.4) recursively. Changing slightly the notation, we set $I_{t+\Delta t}/I_t = \exp(z_{D,t+\Delta t} + J)$, where $J = \ln(j)$ and $z_{D,t+\Delta t} \sim N[(\mu - \lambda\kappa)\Delta t, \sigma_t\sqrt{\Delta t}]$ is the diffusion component. Marginal analysis of borrowing \$1 and investing in the index should in equilibrium yield

$$\begin{aligned}&\mathrm{E}_{\mathrm{t}}\left[I_{t+\Delta t}^{-\varphi}[\exp(z_{D,t+\Delta t} + J - r\Delta t)]\right] = 0 \Rightarrow \\ &\mathrm{E}_{\mathrm{t}}\left[\exp[-\varphi(z_{D,t+\Delta t} + J)]\exp(z_{D,t+\Delta t} + J)\right] = e^{r\Delta t}\mathrm{E}_{\mathrm{t}}\left[\exp[-\varphi(z_{D,t+\Delta t} + J)]\right]\end{aligned}.$$

Since the two random terms are independent, the second line becomes

$$\mathrm{E}_{\mathrm{t}}\left[\frac{\exp[(1-\varphi)z_{D,t+\Delta t}]}{\mathrm{E}_{\mathrm{t}}[\exp(-\varphi z_{D,t+\Delta t})]}\right]\mathrm{E}_{\mathrm{t}}\left[\frac{\exp[(1-\varphi)J)]}{\mathrm{E}_{\mathrm{t}}[\exp(-\varphi J)]}\right] = e^{r\Delta t} = \exp\left[(\mu - \lambda\kappa + \frac{\sigma_t^2}{2} - \varphi\sigma_t^2)\Delta t\right]\mathrm{E}_{\mathrm{t}}\left[\frac{\exp[(1-\varphi)J)]}{\mathrm{E}_{\mathrm{t}}[\exp(-\varphi J)]}\right]$$

from which we get (3.1) as $\Delta t \to 0$ after setting $\exp(-\varphi J) = j^{-\varphi}$, QED.

B. Proof that $\frac{\varphi_{t,T}^{\max}}{\varphi}$ is the highest fraction of option that can be shorted when $\varphi > \varphi_{t.T}^{max}$

Let ζC denotes the maximum quantity of a given overpriced option per unit index held in the IT portfolio that can be shorted by the OT investor at *t* to achieve SA. The return of this short option position, equal to $\zeta(1 + z_{t+\Delta t})$, is invested in proportions β_t^* and $1 - \beta_t^*$ respectively in the index and the riskless asset. As shown in the model free derivation of the SA bounds,[1] the overpriced option position must exceed the value of the expected return of the position with discretized risk neutral dynamics given by (2.4), with $\zeta(1 + z_{t+\Delta t})$ replacing $(1 + z_{t+\Delta t})$. At the continuous time limit of the jump diffusion, however, these risk neutral dynamics tend to (λ^Q, κ^Q) such that $\mu - r = \lambda\kappa - \lambda^Q\kappa^Q$. Since for $\zeta = 1$ and $\varphi > \varphi_{t.T}^{max}$ this equation defines a higher SA upper bound than (3.2), the only way it can hold is if $\zeta\varphi = \varphi_{t.T}^{max}$, QED.

[1] See Perrakis (2019, pp. 23-26).

C. Intermediate Market Structure

The material here is an extension and elaboration of a model presented and available in Perrakis (2019, pp. 232-241), which will only be summarized here.

C.1 Economies of Scale and Market Making in Index Options

To model the MM or DPM in derivative markets we follow the theory of asset allocation under transaction costs and a finite investment horizon, with the horizon interpreted as the length of the review period faced by the MM. The underlying can be an index futures contract or an index-tracking fund. In the presence of frictions continuous time IT portfolio revisions are infeasible. As Constantinides showed in his seminal 1979 article, there exists a no trade (NT) zone for the IT investor, with trades occurring only when the risky asset dynamics bring the value of the risky asset holdings outside the NT zone. Analytical expressions for the derivation of this NT zone and the corresponding optimal IT portfolio policy for a finite horizon *T'* are available only for CRRA utilities and diffusion or jump diffusion asset dynamics, with a numerical algorithm presented in Czerwonko and Perrakis (2016). The algorithm is implemented in discrete time to the discretized dynamics but was shown numerically to converge efficiently to a continuous time limit.

Under competitive conditions assume that an IT investor holds a portfolio with x_t in the riskless asset and a starting long position y_t in an index tracking fund, corresponding to at least one unit of the index at maturity *T'*. $V(x_t, y_t, t)$ denotes the IT value function. To derive it the IT makes sequential investment decisions in the two assets at the discrete trading dates $t = 0,1,...,T'$. The IT portfolio is being revised along the path from *t* to *T'*, with transaction costs 1+*k* for additions to the long and 1-*k* to the short index positions, with no costs for the riskless asset. The SPXW option matures at *T* and the portfolio is liquidated at some value *T'*>*T*. IT liquidates her portfolio by maximizing a concave utility function $U(w_{T'}) = w_{T'}^{1-\alpha} / (1-\alpha)$, and α is the coefficient of relative risk aversion. The terminal condition is $V(x_{T'}, y_{T'}, T') = U(w_{T'}) = U(x_{T'} + y_{T'} - \max[-ky_T, ky_{T'}])$, when all the holdings are converted into cash to form the terminal wealth $w_{T'}$. As an OT she adds an appropriate position in one call option at *t*, which is closed at *T*.

Once the IT model is available and $V(x_t, y_t, t)$ is available for the given relative risk aversion (RRA) and the portfolio (x_t, y_t) we model the option bid and ask quotes as reservation write and reservation purchase prices for an option maturing before the horizon *T'* of the MM acting as IT. The reservation prices are defined as the prices of the short and long option positions that leave the MM indifferent between including or not the option in her portfolio. Hence, they are specific to the particular MM, who also is assumed marginal in the market for an individual option. This assumption implies that the IT portfolio does not change when including the option, as in the entire SA literature. It will be maintained in the following subsection when we examine competitive market making and will be relaxed subsequently for a monopolistic MM. Analytical solution for the option reservation prices are available only under this marginality assumption and rely heavily on the homogeneity property of the value function.[2]

We denote by C_b and C_a the bid and ask prices respectively of a cash-settled European call option maturing at some time *T<T'*, with strike price *K* on the ex-dividend index with current price I_t and let the moneyness $K / I_t \equiv m_t$. Since the MM adopts the opposite option position from an investor, let also $J_q\left(x_t, y_t, t\right),\ q = b, a$ denote the value functions of an MM possessing net long and short positions respectively in n options of that moneyness, and Z_T denote the risky asset return associated with the maturity time *T*. For any date $\tau = t, t+1, ..., T-1$ within the MM horizon the asset dynamics evolve according to the discretized version of the jump-diffusion and the investor revises her portfolio according to $J_q\left(x_t, y_t, t\right) = Max\ E_t\left[J_q\left(x_{t+1}, y_{t+1}, t+1\right)\right]$, where the revisions follow the IT value function $V(x_t, y_t, t)$ by the marginality assumption. We then have the bid and ask prices given respectively by

$$\begin{array}{ll} C_b = \max\{C\},\ \text{such that} & C_a = \min\{C\},\ \text{such that} \\ J_b\left(x_t - nC, y_t, t\right) = V\left(x_t, y_t, t\right), & J_a\left(x_t + nC, y_t, t\right) = V\left(x_t, y_t, t\right). \end{array} \tag{C.1}$$

From this formulation and with the marginality assumption, we get the following two results, proven in Perrakis (2019, pp. 237-239).

Lemma 1: For any MM and for any depth of quote increasing the initial wealth yields a lower (higher) ask (bid) option quote.

Lemma 2: For any MM reducing the depth of the quote yields a lower (higher) ask (bid) option quote.

These, of course, are the definitions of advantages to scale. A financial institution employing market makers and endowing them with more funding than its competitors is able to outbid them in quoting index options without incurring extra risk. These potential economies of scale in option market making, when combined with the informational advantages and the guaranteed minimum share, allow the DPM to protect her

[2] Note also that the marginality assumption is more conservative, insofar as it produces wider option spreads.

monopoly position and are almost certainly responsible for her high share of trading shown earlier in the SPXW market. It also allows large firms, however, to undercut her quotes and compete for specific option positions within the bid-ask spread.

C.2 Market making in SPXW options with futures contracts and under competitive conditions

Under competitive conditions assume that an IT investor holds a portfolio with x_t in the riskless asset and a starting long position y_t in a futures contract, corresponding to at least one unit of the index at maturity *T'*. The SPXW option matures at *T* and the futures at some value *T'*>*T*, the nearest futures maturity time. At *T'*, the IT liquidates her portfolio by maximizing a concave utility function. As an OT, she adds an appropriate position in one call option at *t*, which is closed at *T*. As above, the IT portfolio is being revised along the path from *t* to *T'*, with transaction costs 1+*k* for additions to the long and 1-*k* to the short futures positions, with no costs for the riskless asset. The CRRA assumption is maintained for the IT investor, as well as the marginality assumption, when the IT becomes OT but maintains the same revision policy.

Hedging the IT position with futures is equivalent to setting $\frac{dI_t}{I_t} = \frac{dF_t}{F_t} + \alpha_t dt$ plus a random shock equal to the basis risk. To see this, denote by I_t and F_t the underlying and futures prices at *t* and by Z_{t+1} the ex dividend return $Z_{t+1} = \frac{I_{t+1}e^{-q_t}}{I_t} = \frac{F_{t+1}e^{\psi(t,T')}}{F_t} \Rightarrow \ln(\frac{I_{t+1}e^{-q_t}}{I_t}) = \ln(\frac{F_{t+1}}{F_t}) + \psi(t,T')$, where q_t is the (assumed constant) dividend yield and $\psi(t,T')$ the basis risk, a zero mean independent error term that varies with the distance from futures maturity. At T', $I_{T'} = F_{T'}$ without error. It is clear from the above formulation that the dynamics of the index return can be very closely approximated by the dynamics of the futures return in our case, provided the diffusive volatility is increased to represent the basis risk. Hereafter the basis risk will be ignored in the expressions.

The derivation of the NT zone was done numerically for the CRRA class of utility functions and shown in Czerwonko and Perrakis (2016, equations A.8-A.9) to converge to a continuous time limit when the return $\ln(F_{t+1}/F_t) \equiv z_{t+1}$ converged for both diffusion and jump diffusion. Although we do not need the CRRA assumption for most of our results, we shall assume in what follows that the IT stays in the NT zone for the entire period to option expiration. This is probably a very good approximation for SPXW options, given that the costs for restructuring to the nearest NT boundary are very low in the CRRA case. In such a case, at option expiration T, the value function at t is $V(x_t, y_t, t) = \mathrm{E}_t\left[V(x_t R^{T-t}, y_t z_T, T)\right]$.

Suppose there are $i=1,...,N$ IT investors, each one of whom adopts a short position in one call option with price C, and set $m_t \equiv K/F_t$. Under competitive conditions that eliminate profits we have, if $J^i(x_t, y_t, t)$ denotes the corresponding OT value function for the ith IT investor that the maximum call price that OT would be willing to pay is given by

$$V^i(x_t, y_t, t) = J^i(x_t - C, y_t, t) = E_t[V^i(x_t - C)R^{T-t} + (F_T - K)^+, y_T, T)]. \tag{C.2}$$

Maximizing, we get $F_t \dfrac{E_t[V_x^i(z_T - m_t)^+]}{R^{T-t}E_t[V_x^i]} = C$. An equivalent expression holds for the short option. Since IT traders are marginal in the option market and we have a single period model, the following restrictions hold,[3]

$$C_b \le E_t\left[\frac{V_x^i(T)}{V_x^i(t)}(F_T - K)^+\right] \le C_a. \tag{C.3}$$

Since we know that $V_x(t) \in [\dfrac{V_y(t)}{1+k}, \dfrac{V_y(t)}{1-k}]$ for every t, and letting $\dfrac{V_y^i(T)}{V_y^i(t)} \equiv M^i(z_T)$, a monotone decreasing function, it follows that the OT reservation purchase and write prices are given by

$$C_a \le \frac{1+k}{1-k} Max_{M^i} E_t[M^i(z_T)(F_T - K)^+] \text{ , } C_b \ge \frac{1-k}{1+k} Min_{M^i} E_t[M^i(z_T)(F_T - K)^+]. \tag{C.4}$$

Ignoring the lower bound, which is not empirically relevant and lies always below the SA lower bound, we note that at the continuous time limit, the ask price is bound by the roundtrip transaction costs times the frictionless SA upper bound corresponding to the CRRA IT investor, in which the entire risk premium is used to cover the jump risk. Ignoring the transaction costs, we observe that if the IT equilibrium is in the interior of the frictionless SA bounds, then the ask price is at the SA upper bound. If the RRA is high and the IT equilibrium lies above the SA upper bound then the ask price of that particular IT would still allocate the risk premium entirely to the jump risk, which will lie above the frictionless SA bound. That particular IT investor will not participate in the long option market. We have thus shown that the SA bounds are consistent with perfect competition in the option market even in the presence of frictions in trading the underlying. In such a case we have also confirmed that the competitive frictionless equilibrium lies always inside the reservation aggregate ask and bid prices.

[3] See Constantinides, Jackwerth and Perrakis (2009, pp. 1253-1254).

In fact, even if we set aside information asymmetry and IT wealth, the competitive equilibrium under frictions without a designated MM is much more challenging to model, since each IT trader formulates her own reservation purchase and write prices, dependent on RRA and wealth. A key element of these prices is the depth of the corresponding quote, which is no longer restricted to a single option. Equation (C.1) above has been solved numerically following an adaptation of the Czerwonko and Perrakis (2016a) algorithm for constant volatility jump diffusion dynamics. For a given wealth and under the marginality of the option position assumption it can be shown that the reservation prices are inversely proportional to the depth *n* of the quotes.[4] The market equilibrium can then be a Cournot-style competitive oligopoly, dependent on the observed demand functions of end users for both short and long positions.

Such a competitive environment is also consistent with equal prices for long and short options and with a DPM constrained to be a passive liquidity provider. Let the market consist of competitive end users and a monopolistic liquidity provider. Assume that the total end user demand for a given put option is $N_s = D_s(P_b, P_a, t)$ for short and $N_l = D_l(P_b, P_a, t)$ for long, where the first is increasing in P_b and the second is decreasing in P_a. Assume also, as is reasonable, that $\frac{\partial D_S}{\partial P_a} \geq 0$ and $\frac{\partial D_i}{\partial P_b} \leq 0$. The total net position is $N_l - N_e \equiv N_e > 0$, where N_e is the put market's net exposure, which is on average positive in the OTM region in our data. Then the passive monopolistic market maker is fully hedged in $N_s = D_s(P_b, P_a, t)$ but must cover the net long end user demand with a short position in N_e, which is increasing in both P_b and P_a, but which she must cover at a price of P_b. Exactly the opposite occurs if the end users are net short and the DPM must cover it by purchasing at P_a.

Since this is clearly a suboptimal decision, we now allow the DPM to set freely the prices of the long and short options. Assume as above that the DPM faces a total demand that is net long, with a fully hedged component $N_s = D_s(P_b, P_a, t)$, and residual exposure $N_e(P_b, P_a, t)$, increasing in both arguments. We then have at time *t*, $\hat{x}_t = x_t + N_s(P_a - P_b) + N_e P_b$ and the following maximization problem $Max_{P_a, P_b} J_b\left(\hat{x}_t, y_t, t\right)$, where instead of (3.1) we have

[4] See Perrakis (2019, pp. 238-239, eq. 6.20).

$$J_b\left(\hat{x}_t, y_t, t\right) = E_t[V(\hat{x}_T - N_e F_t(m_t - z_T)^+, y_T, T)] =$$
$$E_t[V(R(x_t + N_s(P_a - P_b) + N_e P_b) - N_e F_t(m_t - z_T)^+, y_T, T)] \quad \text{(C.5)}$$

Maximizing $J_b\left(\hat{x}_t, y_t, t\right)$ with respect to P_a and P_b we get after simplification

$$F_t \frac{E_t[V_x(m_t - z_T)^+]}{RE_t[V_x]} = \frac{1}{\frac{\partial N_e}{\partial P_a}}[N_s + \frac{\partial N_s}{\partial P_a}(P_a - P_b) + P_b \frac{\partial N_e}{\partial P_a}]$$
$$F_t \frac{E_t[V_x(m_t - z_T)^+]}{RE_t[V_x]} = \frac{1}{\frac{\partial N_e}{\partial P_b}}[-N_s + \frac{\partial N_s}{\partial P_b}(P_a - P_b) + P_b \frac{\partial N_e}{\partial P_b}] \quad \text{(C.6)}$$

If P_a and P_b are constrained to be equal to P then it can be shown that the right-hand-side of both expressions in (3.3) is equal to $P(1+e_N^{-1})$, where $e_N > 0$ is the elasticity of residual demand $N_e(P,t)$. This appears perverse, since the DPM charges less than P for the short position, but on the other hand it reduces the fully hedged position, from which she receives no income. It also justifies partially the observed consistency of the option market data with the SA bounds at the left tail. Conversely, the elasticity is negative when the DPM exposure is net long, as it mostly is in the ATM zone for both calls and puts. In both cases the bid-ask spread raises the prices. In either case the equilibrium prices depend on the demand elasticity, varying inversely with $\left|e_N\right|$. They do not depend on the asset dynamics and are specific to the degree of moneyness. Hence, they can accommodate the differing distances between the SA bounds and the observed bid-ask midpoint in our results since there is no reason for the demand forces to be the same along the cross section.[5]

The above equilibrium can be extended with very little reformulation to a Cournot-style oligopoly, which is most probably the prevailing market structure in SPXW. In Table OA5 it is shown clearly that in the key ATM zones for both calls and puts the average observed DPM market share is at or above 50%, which is what would be the DPM entitlement when facing a duopoly. In fact, it may actually be the optimally chosen quantity of Cournot players facing the demand and supply functions of the retail investors.

[5] In fact, AFT point out in several places (e.g., p. 1336) that the pricing rules differ between OTM and ATM options, as shown also in our data.

Table OA1: Jump Diffusion Parameter Estimates

Parameter	Estimate	Std Err	t Value	Approx Pr > |t|
μ	0.0707	0.0234	3.03	0.0025
σ	0.1497	0.0031	48.40	<.0001
μ_j	-0.0322	0.0164	-1.96	0.0497
σ_j	0.1007	0.0107	9.42	<.0001
λ	0.2625	0.1572	1.67	0.095

This table presents parameter estimates for the the jump diffusion model obtained from the GMM procedure using daily returns of the S&P 500 index from 1963 to 2010.

Table OA2: Summary Statistics Weeklys - All Cross Sections

	All	DTM1	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9
Panel A: Summary Statistics Calls								
# Contracts	140,305	17,170	18,769	19,601	20,059	22,290	21,652	20,764
Avg IV	22.7%	32.0%	25.7%	22.6%	21.2%	21.4%	20.1%	19.0%
Avg Moneyness (K/S)	0.98	0.96	0.97	0.98	0.98	0.99	0.99	0.99
Avg Mid-Quotes	94.5	128.8	115.5	94.3	90.5	89.6	77.2	74.6
Relative BA Spread	8.4%	6.3%	7.6%	8.8%	10.0%	10.3%	7.9%	7.6%
Option Bounds Spread	10.6%	3.5%	6.2%	8.8%	10.8%	12.6%	14.5%	15.7%
Avg NetBuy MM	4	-13	-1	-0.2	0.2	25	9	3
% Volume MM	51%	51%	52%	52%	51%	50%	51%	50%
Total Volume	101,788	24,412	16,716	13,562	13,327	15,058	10,744	7,970
Total Open Interest	265,293	33,565	36,738	40,007	39,723	39,732	37,882	37,646
Panel B: Calls Mispricing vs. SA Bounds								
Mid-Quotes Within Bounds	33,128	3,687	4,495	4,985	5,345	5,430	4,776	4,410
	24%	21%	24%	25%	27%	24%	22%	21%
Overpriced Contracts	51,880	4,866	6,397	6,947	6,334	8,764	9,662	8,910
	37%	28%	34%	35%	32%	39%	45%	43%
Cross-sections >1 Overpriced	2,807	412	404	411	345	415	428	392
	90%	93%	90%	90%	81%	90%	93%	92%
Cross-sections >25% Overpriced	2,274	251	305	330	285	369	385	349
	73%	56%	68%	72%	67%	80%	83%	82%
Cross-sections >50% Overpriced	1,279	88	153	167	152	217	260	242
	41%	20%	34%	37%	36%	47%	56%	57%
Panel C: Summary Statistics Puts								
# Contracts	194,470	15,423	23,284	27,455	29,072	33,461	33,320	32,455
Avg IV	23.4%	24.5%	23.8%	23.1%	23.1%	23.9%	23.4%	22.4%
Avg Moneyness	0.98	1.01	0.99	0.98	0.98	0.97	0.97	0.96
Avg Mid-Quotes	37.2	60.2	45.2	34.4	38.9	34.6	30.8	30.4
Relative BA Spread	12.4%	10.1%	16.6%	16.6%	14.9%	11.4%	9.2%	9.2%
Option Bounds Spread	77.2%	39.4%	65.4%	75.4%	78.1%	84.4%	86.5%	87.6%
Avg NetBuy MM	5	-15	-13	-8	4	46	-2	1
% Volume MM	50%	51%	51%	50%	49%	48%	49%	49%
Total Volume	188,261	34,302	31,262	26,684	27,724	34,315	19,832	14,142
Total Open Interest	482,479	38,406	71,039	82,927	77,925	74,668	69,274	68,240
Panel D: Puts Mispricing vs. SA Bounds								
Mid-Quotes Within Bounds	75,122	2,774	7,404	10,314	11,793	14,122	14,136	14,579
	39%	18%	32%	38%	41%	42%	42%	45%
Overpriced Contracts	68,378	5,642	7,693	8,746	8,344	12,096	13,480	12,377
	35%	37%	33%	32%	29%	36%	40%	38%
Cross-sections >1 Overpriced	2,853	419	409	413	365	418	433	396
	91%	94%	91%	90%	85%	91%	94%	93%
Cross-sections >25% Overpriced	2,250	389	329	317	231	313	353	318
	72%	87%	73%	69%	54%	68%	76%	75%
Cross-sections >50% Overpriced	785	138	111	92	66	125	144	109
	25%	31%	25%	20%	15%	27%	31%	26%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes are at 3:00 PM from 20110317 to 20220317 for the total of 477 expiring Fridays and 3,127 observation days. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped based on the number of days-to-maturity (DTM), from 9 days to 1 day. "Relative BA Spread" ("Option Bounds Spread") is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). "Avg NB MM" is the average number of contracts purchased minus sold by market makers. "Total Volume" and "Total Open Interest" are in thousands. "Cross-sections >25% Overpriced" shows number of cross sections where more than 25% of contracts in the cross section are overpriced.

Table OA3: Summary Statistics Weeklys - All Options

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	8,170	6,415	17,427	24,783	31,476	25,786	13,847	6,480	5,921	140,305
% Contracts	5.8%	4.6%	12.4%	17.7%	22.4%	18.4%	9.9%	4.6%	4.2%	100%
Avg IV	80.3%	38.8%	28.1%	20.9%	16.3%	14.6%	16.7%	20.7%	33.0%	22.7%
Avg Mid-Quotes	609.6	255.5	158.0	90.1	40.3	11.3	4.3	2.9	2.6	94.5
Relative BA Spread	1.5%	2.0%	2.9%	3.7%	3.6%	6.0%	17.9%	32.1%	49.8%	8.4%
Option Bounds Spread	0.1%	0.5%	0.9%	1.9%	5.0%	18.6%	30.3%	32.4%	26.0%	10.6%
Avg NetBuy MM	-0.1	1	-5	-14	4	24	26	1	-13	4
% Volume MM	62%	51%	51%	49%	51%	51%	50%	49%	51%	51%
Total Volume	127	123	609	1,759	22,052	57,979	13,826	3,455	1,859	101,788
Total Open Interest	724	1,842	10,029	28,984	85,911	91,711	29,024	9,844	7,224	265,293
Panel B: Calls Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	2,011	3,335	8,929	9,826	4,403	1,533	1,763	774	554	33,128
	25%	52%	51%	40%	14%	6%	13%	12%	9%	24%
Overpriced Contracts	334	542	2,372	7,350	20,298	15,149	3,668	1,029	1,138	51,880
	4%	8%	14%	30%	64%	59%	26%	16%	19%	37%
Cross-sections >1 Overpriced	83	134	478	1,340	2,733	2,456	769	156	76	2,807
	4%	7%	17%	43%	87%	80%	45%	20%	18%	90%
Cross-sections >50% Overpriced	44	117	327	858	2,146	2,236	644	134	55	1,279
	2%	6%	11%	28%	69%	73%	38%	17%	13%	41%
Cross-sections 100% Overpriced	22	106	234	593	1,330	2,015	550	117	47	26
	1%	5%	8%	19%	43%	66%	32%	15%	11%	1%
Panel C: Summary Statistics Puts										
# Contracts	10,262	26,944	38,921	29,219	31,201	28,978	13,793	6,103	9,049	194,470
% Contracts	5.3%	13.9%	20.0%	15.0%	16.0%	14.9%	7.1%	3.1%	4.7%	100%
Avg IV	41.4%	31.1%	25.2%	20.3%	16.3%	14.0%	18.4%	25.3%	47.9%	23.4%
Avg Mid-Quotes	2.7	1.7	2.7	5.5	12.5	37.6	86.5	145.2	368.0	37.2
Relative BA Spread	18.3%	25.6%	20.6%	11.2%	5.0%	5.1%	4.7%	3.6%	2.5%	12.4%
Option Bounds Spread	156.1%	148.8%	128.0%	98.2%	47.3%	2.6%	0.4%	0.2%	0.0%	77.2%
Avg NetBuy MM	3	-19	-4	45	0	12	-2	-3	-7	5
% Volume MM	47%	45%	48%	49%	52%	51%	49%	50%	48%	50%
Total Volume	3,214	13,533	33,585	47,364	71,141	16,902	1,556	534	433	188,261
Total Open Interest	14,630	64,515	125,974	108,641	102,448	40,460	12,743	5,704	7,364	482,479
Panel D: Puts Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	7,168	19,968	24,935	14,046	4,953	1,839	1,339	501	373	75,122
	70%	74%	64%	48%	16%	6%	10%	8%	4%	39%
Overpriced Contracts	2,865	4,667	9,869	13,550	22,405	13,045	1,383	327	267	68,378
	28%	17%	25%	46%	72%	45%	10%	5%	3%	35%
Cross-sections >1 Overpriced	204	388	902	1,832	2,765	2,468	301	55	25	2,853
	18%	18%	34%	63%	88%	79%	11%	4%	3%	91%
Cross-sections >50% Overpriced	189	319	628	1,332	2,505	1,512	118	34	3	785
	17%	15%	23%	46%	80%	48%	4%	2%	0%	25%
Cross-sections 100% Overpriced	178	289	470	979	1,864	497	75	25	3	4
	16%	13%	18%	34%	60%	16%	3%	2%	0%	0%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed at 3:00 PM from 20110317 to 20220317, expiring on Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. "Relative BA Spread" ("Option Bounds Spread") is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). "Avg NB MM" is the average number of contracts purchased minus sold by market makers. "Total Volume" and "Total Open Interest" are in thousands. "Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table OA4: Bid-Ask Quotes vs Option Bounds - All Cross Sections

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Call Mid-Quotes Within Option Bounds										
DTM1-Thu	193	348	1029	1318	626	128	38	6	1	3687
	13%	32%	36%	35%	14%	5%	6%	5%	1%	21%
DTM2-Wed	327	495	1243	1461	665	135	151	16	2	4495
	21%	45%	47%	40%	15%	4%	12%	4%	1%	24%
DTM3-Tue	268	514	1471	1602	616	200	210	53	51	4985
	24%	55%	58%	44%	13%	6%	12%	7%	9%	25%
DTM4-Mon	313	529	1487	1731	642	279	216	101	47	5345
	29%	61%	60%	50%	15%	7%	10%	10%	5%	27%
DTM7-Fri	405	622	1514	1438	621	248	358	137	87	5430
	34%	64%	56%	39%	14%	6%	14%	11%	7%	24%
DTM8-Thu	283	431	1181	1164	610	300	429	225	153	4776
	31%	57%	54%	34%	13%	7%	15%	16%	11%	22%
DTM9-Wed	222	396	1004	1112	623	243	361	236	213	4410
	29%	58%	49%	35%	14%	6%	13%	16%	15%	21%
Panel B: Call Overpricing Given Option Bounds										
DTM1-Thu	13	15	78	384	2,353	1,665	246	45	67	4,866
	1%	1%	3%	10%	52%	61%	41%	41%	87%	28%
DTM2-Wed	38	61	195	737	2,720	1,936	389	145	176	6,397
	2%	6%	7%	20%	61%	58%	30%	35%	66%	34%
DTM3-Tue	38	56	213	867	3,034	2,092	404	130	113	6,947
	3%	6%	8%	24%	66%	58%	23%	17%	19%	35%
DTM4-Mon	37	54	234	776	2,470	1,873	426	163	301	6,334
	3%	6%	9%	23%	57%	50%	19%	16%	33%	32%
DTM7-Fri	49	104	483	1,411	3,202	2,526	634	172	183	8,764
	4%	11%	18%	38%	70%	62%	26%	13%	14%	39%
DTM8-Thu	82	127	545	1,646	3,379	2,646	824	227	186	9,662
	9%	17%	25%	48%	74%	63%	30%	16%	14%	45%
DTM9-Wed	77	125	624	1,529	3,140	2,411	745	147	112	8,910
	10%	18%	30%	48%	71%	59%	27%	10%	8%	43%
Panel C: Put Mid-Quotes Within Option Bounds										
DTM1-Thu	0	0	624	1,160	668	172	109	23	18	2,774
	0%	0%	68%	53%	17%	4%	5%	2%	1%	18%
DTM2-Wed	0	973	2,866	2,346	786	201	141	39	52	7,404
	0%	76%	61%	55%	17%	5%	7%	4%	4%	32%
DTM3-Tue	236	2,255	4,110	2,403	760	271	191	58	30	10,314
	62%	70%	64%	52%	16%	6%	10%	7%	2%	38%
DTM4-Mon	801	3,079	4,123	2,388	706	339	210	91	56	11,793
	71%	67%	64%	55%	16%	8%	11%	10%	4%	41%
DTM7-Fri	1,637	4,387	4,642	2,104	699	274	233	90	56	14,122
	70%	74%	67%	45%	15%	6%	11%	9%	4%	42%
DTM8-Thu	1,935	4,619	4,383	1,804	680	310	220	106	79	14,136
	67%	77%	63%	39%	15%	7%	11%	13%	7%	42%
DTM9-Wed	2,559	4,655	4,187	1,841	654	272	235	94	82	14,579
	74%	79%	63%	41%	15%	7%	13%	13%	8%	45%
Panel D: Put Overpricing Given Option Bounds										
DTM1-Thu	9	35	222	905	2,883	1,460	114	14	0	5,642
	100%	100%	24%	41%	74%	36%	6%	1%	0%	37%
DTM2-Wed	50	148	817	1,592	3,206	1,649	157	41	33	7,693
	100%	12%	17%	37%	71%	40%	8%	5%	2%	33%
DTM3-Tue	136	403	988	1,853	3,354	1,810	151	32	19	8,746
	36%	13%	15%	40%	72%	43%	8%	4%	2%	32%
DTM4-Mon	296	627	1,197	1,582	2,763	1,502	193	67	117	8,344
	26%	14%	19%	37%	64%	38%	10%	8%	8%	29%
DTM7-Fri	608	1,169	1,997	2,392	3,373	2,194	270	55	38	12,096
	26%	20%	29%	51%	73%	51%	12%	6%	3%	36%
DTM8-Thu	888	1,213	2,371	2,708	3,555	2,355	293	69	28	13,480
	31%	20%	34%	58%	76%	56%	15%	8%	3%	40%
DTM9-Wed	878	1,072	2,277	2,518	3,271	2,075	205	49	32	12,377
	25%	18%	34%	56%	73%	52%	12%	7%	3%	38%

Panels A and C report number and percentage of option contracts with the bid-ask quotes between option bounds. Panels B and D report number and percentage of overpriced option contracts with the ask price above option upper bounds. Statistics are grouped based on the number of days-to-maturity (DTM), from 9 days to 1 day.

Table OA5: Volume and Net Buy Market Makers

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Call Percentage Volume MM and Total Volume (x1000)										
DTM1-Thu	77%	48%	52%	48%	51%	52%	51%	49%	53%	51%
	39	24	104	365	6,512	15,352	1,782	163	71	24,412
DTM2-Wed	51%	52%	48%	50%	51%	52%	51%	50%	50%	52%
	15	16	96	286	3,949	9,944	1,931	371	108	16,716
DTM3-Tue	58%	48%	52%	49%	52%	52%	50%	51%	51%	52%
	18	18	84	258	3,048	7,739	1,766	410	221	13,562
DTM4-Mon	54%	56%	49%	47%	52%	51%	49%	47%	50%	51%
	12	13	103	269	2,400	7,351	2,177	666	337	13,327
DTM7-Fri	60%	52%	52%	51%	51%	50%	50%	46%	50%	50%
	16	17	107	259	2,562	7,871	2,790	926	509	15,058
DTM8-Thu	57%	51%	53%	50%	52%	50%	51%	51%	54%	51%
	19	22	59	189	2,016	5,653	1,864	559	364	10,744
DTM9-Wed	53%	50%	51%	54%	51%	50%	50%	50%	51%	50%
	9	13	55	133	1,565	4,070	1,515	361	249	7,970
Panel B: Call Net Buy MM										
DTM1-Thu	-2	0	-2	-27	-21	-4	17	-122	2	-13
DTM2-Wed	0	2	-4	-12	14	30	-55	-77	-80	-1
DTM3-Tue	1	0	-2	-19	9	42	-3	-68	-108	0
DTM4-Mon	0	2	-5	-18	4	23	8	-26	-22	0
DTM7-Fri	2	3	-8	-10	14	37	112	87	8	25
DTM8-Thu	0	2	-9	-6	7	29	22	18	-5	9
DTM9-Wed	0	1	-6	-7	-3	7	24	-8	16	3
Panel C: Put Percentage Volume MM and Total Volume (x1000)										
DTM1-Thu	52%	53%	52%	51%	52%	52%	48%	49%	47%	51%
	10	60	1,078	6,000	21,566	5,070	361	107	50	34,302
DTM2-Wed	55%	50%	49%	50%	52%	52%	49%	51%	47%	51%
	18	563	4,367	9,264	13,738	2,947	253	51	62	31,262
DTM3-Tue	54%	47%	48%	49%	52%	52%	48%	49%	44%	50%
	87	1,021	5,288	7,738	9,834	2,406	177	79	56	26,684
DTM4-Mon	50%	44%	48%	48%	52%	52%	50%	50%	50%	49%
	387	2,338	6,878	7,737	8,007	1,973	253	81	70	27,724
DTM7-Fri	48%	43%	47%	49%	50%	50%	50%	50%	49%	48%
	1,058	5,209	8,337	8,496	8,691	2,143	215	80	87	34,315
DTM8-Thu	45%	44%	49%	48%	52%	51%	52%	53%	52%	49%
	799	2,513	4,370	4,919	5,556	1,389	164	74	47	19,832
DTM9-Wed	44%	46%	48%	50%	51%	51%	49%	46%	45%	49%
	855	1,829	3,268	3,211	3,749	974	134	62	61	14,142
Panel D: Put Net Buy MM										
DTM1-Thu	-393	-126	-26	104	-66	-25	-15	-10	-15	-15
DTM2-Wed	36	-90	-102	35	16	18	1	-8	-4	-13
DTM3-Tue	-34	-26	-51	78	-28	2	-3	-8	-8	-8
DTM4-Mon	3	-44	11	65	2	-4	-6	-8	-6	4
DTM7-Fri	35	-21	96	84	72	44	8	3	-10	46
DTM8-Thu	-13	9	0	-27	-17	28	11	-3	-3	-2
DTM9-Wed	-1	-8	-9	10	10	16	-12	15	-6	1

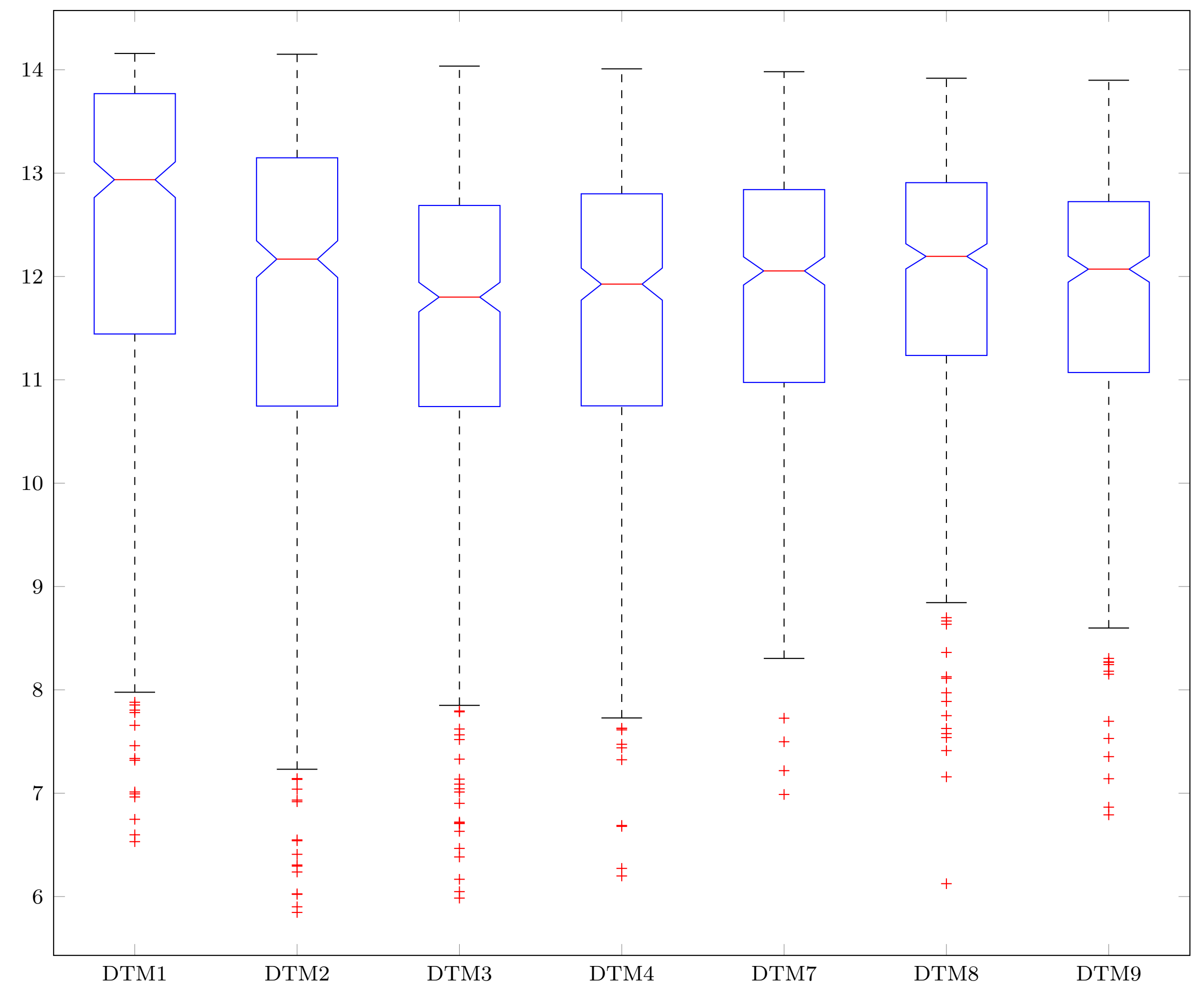
Panels A and C report percentage of volume by market markers and the total trading volume for calls and puts. Panels B and D reports the average net buy by market makers. Options are grouped based on the number of days-to-maturity (DTM), from 9 days to 1 day and statistics are provided across moneyness bins. The option data are based on the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes are at 3:00 PM from 20110317 to 20220317 for the total of 477 expiring Fridays and 3,127 observation days. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents.

Table OA6: Return Characteristics Option Trader Portfolios

	Panel A: Open @Bid - Close @UB						Panel B: Open @Bid - Close @Ask						Panel C: Open @Midpoint - Close @Midpoint					
	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9
Calls:																		
Avg Return	28.7%	16.6%	17.2%	23.7%	29.4%	25.9%	7.1%	-8.7%	-10.0%	4.1%	-2.4%	-7.5%	13.2%	-3.0%	-4.6%	9.9%	3.8%	-2.5%
St. Dev.	6.8%	7.0%	5.9%	6.8%	6.2%	5.2%	6.7%	6.9%	6.0%	6.4%	5.8%	4.9%	6.5%	6.7%	5.9%	6.4%	5.7%	4.8%
Info Ratio	4.20	2.36	2.91	3.47	4.78	4.95	1.06	-1.26	-1.66	0.64	-0.41	-1.54	2.02	-0.45	-0.78	1.55	0.66	-0.53
H_0: OT $\not\succ_2$ IT	0	0.004	0.001	0	0	0	0.082	1	1	0.206	1	1	0.004	1	1	0.044	0.189	1
H_0: OT $\not\succ_2$ IT (5%)	0	0	0	0	0	0	0.010	1	1	0.011	1	1	0.001	1	1	0	0.009	1
H_0: OT $\not\succ_2$ IT (10%)	0	0	0	0	0	0	0	1	1	0	1	1	0	1	1	0	0	1
% CS Ret>0	62%	62%	52%	60%	64%	60%	52%	48%	39%	50%	50%	42%	56%	51%	41%	53%	53%	44%
No. CS	382	387	318	361	399	354	382	387	318	361	399	354	382	387	318	361	399	354
No. Contracts	3,868	4,451	3,698	4,944	5,615	5,170	3,868	4,451	3,698	4,944	5,615	5,170	3,868	4,451	3,698	4,944	5,615	5,170
Puts:																		
Avg Return	22.6%	13.9%	12.7%	29.1%	30.3%	28.6%	2.7%	-9.3%	-12.3%	6.1%	-4.7%	-7.6%	7.2%	-5.3%	-8.5%	11.4%	0.7%	-3.2%
St. Dev.	7.7%	8.3%	7.2%	8.4%	7.7%	6.2%	7.0%	7.5%	6.7%	7.4%	6.6%	5.2%	7.1%	7.6%	6.7%	7.6%	6.7%	5.3%
Info Ratio	2.95	1.68	1.76	3.45	3.94	4.62	0.38	-1.23	-1.84	0.82	-0.71	-1.45	1.02	-0.70	-1.26	1.50	0.10	-0.61
H_0: OT $\not\succ_2$ IT	0	0.019	0.013	0	0	0	0.308	1	1	0.149	1	1	0.108	1	1	0.027	0.445	1
H_0: OT $\not\succ_2$ IT (5%)	0	0	0.005	0	0	0	0.133	1	1	0.014	1	1	0.024	1	1	0	0.099	1
H_0: OT $\not\succ_2$ IT (10%)	0	0	0	0	0	0	0.040	1	1	0	1	1	0.001	1	1	0	0.006	1
% CS Ret>0	55%	54%	47%	54%	59%	55%	49%	47%	40%	48%	49%	42%	51%	47%	41%	50%	51%	43%
No. CS	383	387	332	362	404	353	383	387	332	362	404	353	383	387	332	362	404	353
No. Contracts	4,158	4,879	3,967	4,925	5,712	5,147	4,158	4,879	3,967	4,925	5,712	5,147	4,158	4,879	3,967	4,925	5,712	5,147

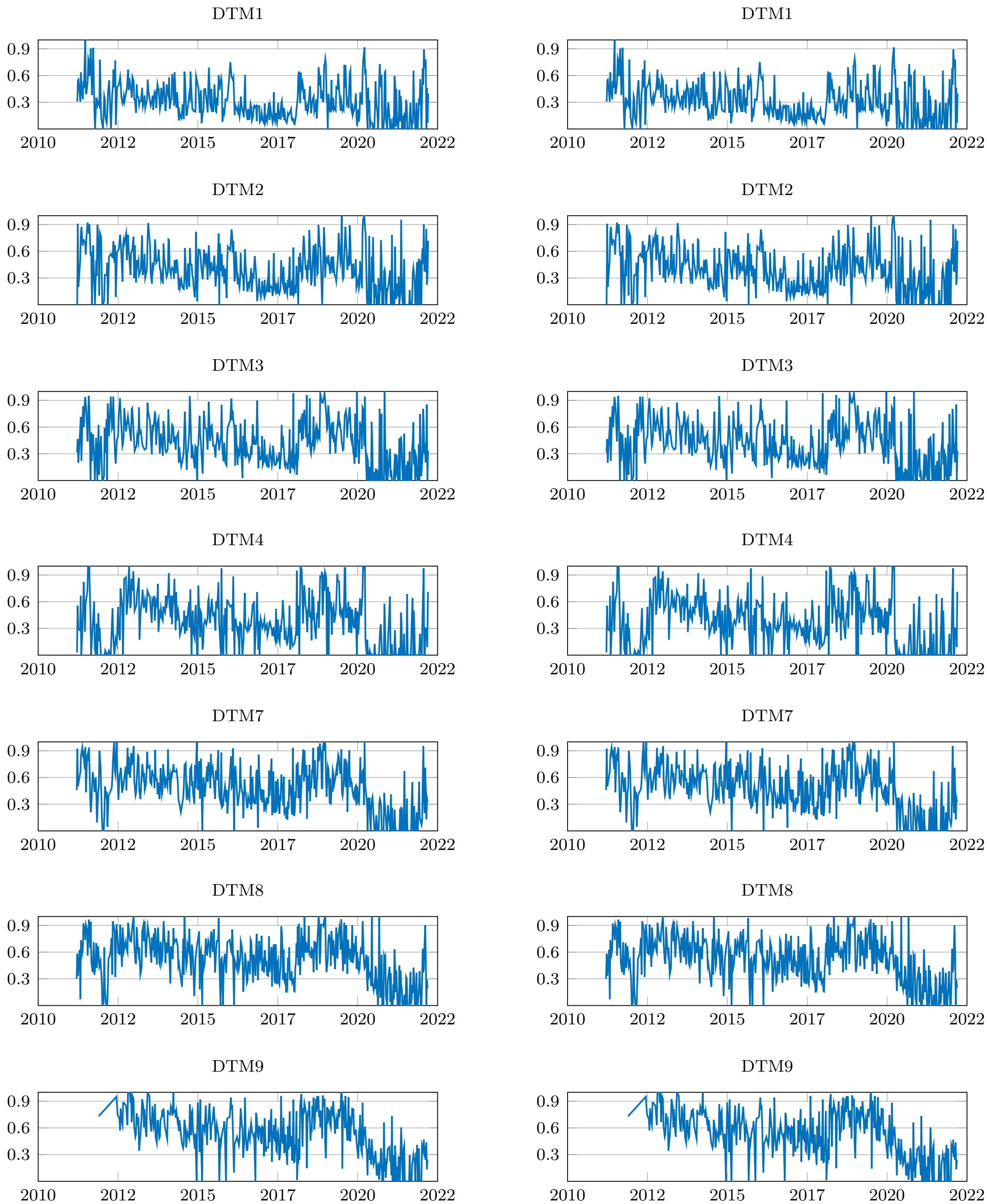
The table reports statistical properties of excess returns of option trader (OT) portfolios, including arithmetic average daily returns (annualized) and standard deviations. The row “% CS Ret>0" shows the percentage of weeks (trades) with positive portfolio returns. The returns in Panels A to B are computed when OT writes an overpriced option at its bid quote and closes her position at the option upper bound and at the ask quote. Panels C reports similar results when OT open and close her position at the midpoint of bid and ask quotes. The trading strategy is implemented by using 2% ATM options, end-of-week expiration, with non-zero volume in two consecutive days. Column DTM2 shows statistics when the portfolio is set using options with two days-to-maturity on Wednesday and the positions are closed at the next trading day. The OT portfolios are across all weeks in the sample and when there is no overpriced ATM option, OT=IT. The table also reports p-values for Davidson-Duclos (2007) second order stochastic dominance test for paired (correlated) outcomes.The row H_0: OT $\not\succ_2$ reports p-values for the null of non-dominance of time series of OT portfolio returns over time series of IT portfolio returns, with no trimming in the right tail (fourth row), 5% trimming in the right tail (fifth row), and 10% trimming in the right tail (six row).

Figure OA1: SA Implied Upper Bound RRA



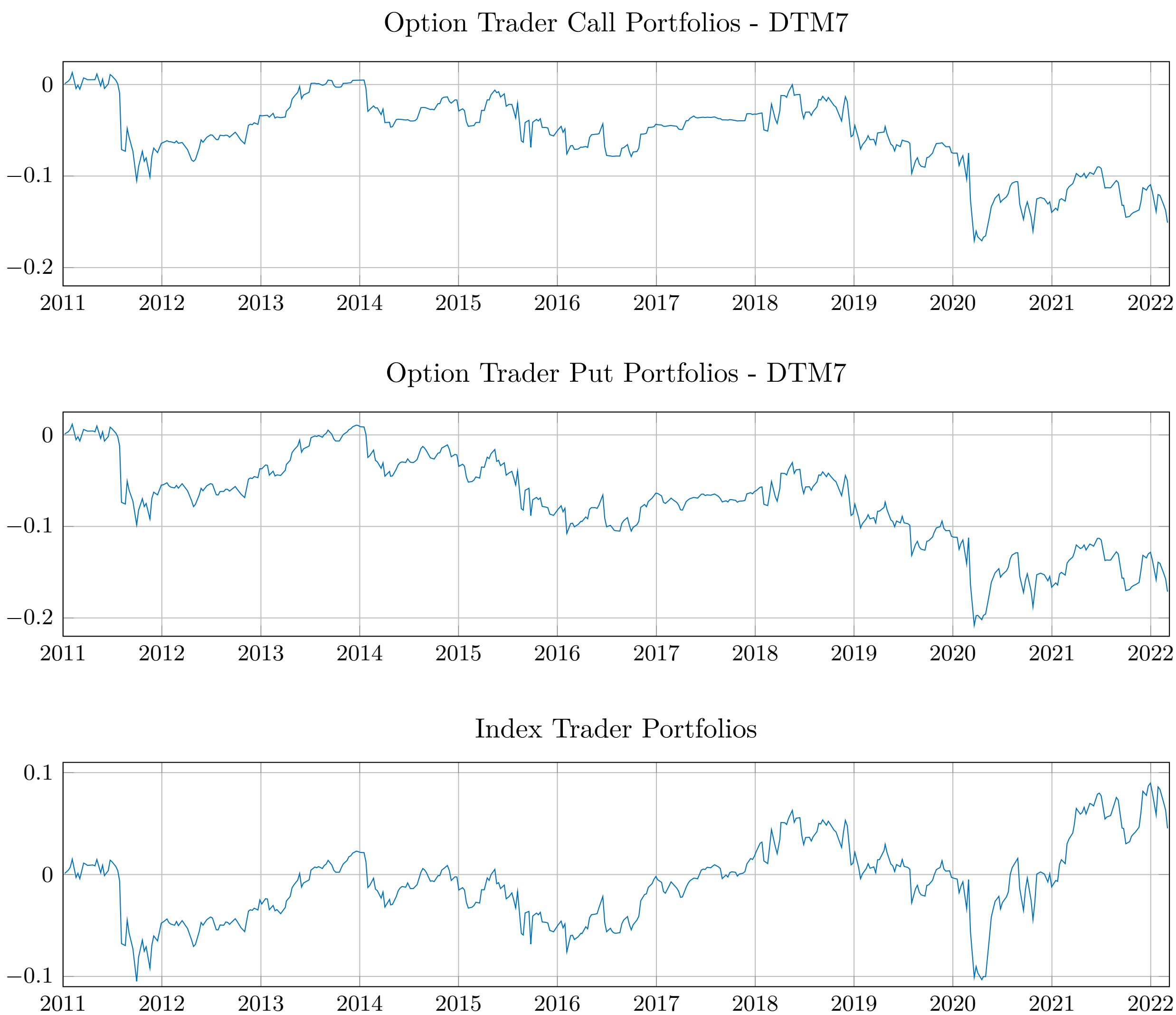
This figure shows the distribution of relative risk aversions implied by stochastic arbitrage upper bounds on option prices over the entire sample. The implied RRAs are reported in groups based on number of day-to-maturity. The results are obtained for weekly options, end-of-week expiration, with nonzero volume in two consecutive trading days.

Figure OA2: Percentage Overpriced Options



This figure shows the percentage overpriced call (left panel) and put (right panel) contracts with respect to the stochastic arbitrage upper bounds on option prices. The statistics are obtained for weekly options, end-of-week expiration, with nonzero trading volume in two consecutive trading days.

Figure OA3: 1-day Log-Return - All but Overpriced Options



The figure shows cumulative one-day returns for an option trader portfolio when OT trades all but overpriced options identified by the SA approach. The trading strategy is implemented by using 2% ATM options, end-of-week expiration, with non-zero volume in two consecutive days. The bottom panel plots the cumulative return of an index trader.